Product Differentiation and Relative Performance Evaluation in an Asymmetric Duopoly.

Aditi Sengupta.*
Southern Methodist University, Dallas, Texas.

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Abstract

In a model of managerial delegation in a duopoly with asymmetric costs, I show that an increase in the intensity of market competition (product differentiation) increases the absolute weight placed on rival's profit (relative performance) in the managerial compensation scheme for both firms and also increases market concentration. The relatively efficient (larger) firm always places higher weight on rival's performance and obtains higher market share.

Key-words: Strategic Delegation; Relative Performance; Managerial Compensation; Oligopoly.

JEL Classification: D43; L13; M52; M21.

*Department of Economics, Southern Methodist University, 3300 Dyer Street, Dallas, TX 75275-0496. E-mail: asengupt@smu.edu.

1 Introduction

A substantial body of literature has studied the strategic incentives of competing firms to delegate decision making on market variables (such as pricing or output) to managers. In such situations, the outcome of market competition is determined by the managerial compensation scheme.[1] One strand of this literature has focused on compensation schemes that depend on a firm's own performance (profit) and the performance of rival firms. The use of such compensation schemes corresponds to relative performance evaluation of executives.

In their seminal paper, Aggarwal and Samwick (1999) show that in a symmetric two stage duopoly where firms determine the relative weights on own profit and rival's profit in their managerial compensation prior to market competition (in prices or output), the equilibrium is always one where firms put some weight on rival's performance. This weight is negative in the case of quantity competition and positive for price competition. Relative to the outcome with no delegation, the market outcome is more competitive in the former case and more collusive in the latter case. They show that the sensitivity of executive compensation to relative performance evaluation (i.e., the absolute weight placed on rival's profit) is increasing in the intensity of market competition as measured by the degree of product differentiation. The authors find empirical support for this key conjecture.

This note extends the theoretical analysis in Aggarwal and Samwick (1999) to a duopoly with asymmetric cost and characterizes the effect of change in product differentiation on the sensitivity of executive compensation to rival firm's profit (or relative performance) and on the market outcome.

We find that (irrespective of the extent of product differentiation), executive compensation in the relatively efficient firm, which also acquires higher market share (larger size), is always more sensitive to the rival firm's profit than the relatively inefficient (smaller) firm . An increase in the intensity of market competition (decrease in the degree of product differentiation) increases the absolute values of the weights placed on rival's profit for both the efficient and the inefficient firms and further, magnifies the asymmetry in market shares between the firms i.e., increases market concentration. None of these qualitative results depend on whether firms compete in quantities or prices (i.e., strategic complementarity or substitutability of competitive variables). In the case of quantity competition, strategic delegation accen-

[1] See, among others, Vickers(1985), Fershtman and Judd (1987).

tuates the asymmetry between firms in terms of market shares relative to the benchmark case (no delegation).

Miller and Pazgal (2002) also analyze a similar model to this note; however, their analysis of the asymmetric cost case assumes that there the products are not differentiated.

2 Model

We consider a market with two firms that sell horizontally differentiated products. Each firm delegates the task of determining its output and price in the market to its manager. The firm i offers a linear incentive contract of the following form to her manager

$$w_i = \gamma_{oi} + \gamma_{1i}\left[\alpha_i \pi_i + (1-\alpha_i)(\pi_i - \pi_j)\right], \ \ \gamma_{1i} > 0, \ i,j = 1,2, \ i \neq j, \quad (1)$$

where w_i is the wage earned by the manager of the firm i and π_i is the profit of the firm i. We consider a two stage simultaneous move game. In the first stage, each firm i chooses $\alpha_i \in R$ i.e. , the weights on her own profit(π_i) and relative profit $(\pi_i - \pi_j)$ in the linear incentive contract. In the next stage with the knowledge of these weights the managers compete in the market either in quantities or in prices. The appropriate values of the parameters $(\gamma_{0i}, \gamma_{1i})$ are selected such that in the equilibrium of the two stage game $w_i = \overline{w}$ *where* $\overline{w} > 0$ is the reservation wage of the manager. Note that in the second stage of the game the objective function of the manager of firm i reduces to

$$\alpha_i \pi_i + (1-\alpha_i)(\pi_i - \pi_j) = \pi_i - (1-\alpha_i)\pi_j, \quad i,j = 1,2, \ i \neq j. \quad (2)$$

In the next two sections we consider two versions of this model, one in which firms compete in quantities and the other in which firms engage in price competition in the second stage.

3 Quantity Competition

In this section we discuss a model of quantity competition (a differentiated Cournot model). Each firm faces an inverse demand function

$$p_i(q_i, q_j) = A - q_i - \mu q_j, \quad i,j = 1,2 \ \ i \neq j \text{ and } \ 0 < \mu \leq 1, \quad (3)$$

where reciprocal of μ is the degree of product differentiation. The cost function of the firm i is given by

$$C_i(q_i) = c_i q_i^2 \ , \ i = 1,2. \quad (4)$$

Our assumption of upward sloping marginal cost differs from the cost function in the symmetric duopoly analyzed by Aggarwal and Samwick (1999) where firms are assumed to produce under *constant* returns to scale. The reason why we choose to depart from the assumption of constant returns is that when unit costs of production are constant and differ between the firms, a subgame perfect Nash equilibrium in pure strategies for the two stage game does not exist (the reaction functions in the reduced form first stage game are parallel curves in the relevant range).

Assume $c_1 < c_2$ i.e., firm 1 has lower marginal cost for every level of output and both the firms have zero marginal costs at zero. To solve the simultaneous move game by backward induction first we consider the maximization problem of the manager of the firm i

$$\begin{aligned} & \max_{q_i}\{\alpha_i\pi_i + (1-\alpha_i)(\pi_i - \pi_j)\} \\ = \ & \max_{q_i}\{q_i(A - q_i - \mu q_j - c_i q_i^2) - (1-\alpha_i)q_j(A - \mu q_i - q_j - c_j q_j^2)\}, \ i,j = 1,2 \ ,i \neq j. \end{aligned} \tag{5}$$

This yields the following reaction function in the second stage game

$$q_i = \max\{\frac{A}{2\,(1+c_i)} - \frac{\alpha_i\mu}{2\,(1+c_i)}q_j, 0\}, \ i,j = 1,2, \ i \neq j. \tag{6}$$

If $\alpha_i\mu < 2\,(1+c_i)$ for $i = 1,2$ then the unique Nash equilibrium of this second stage game is

$$q_i^*\,(\alpha_i,\alpha_j) = \frac{A(2(1+c_i) - \alpha_i\mu)}{4(1+c_i+c_j+c_ic_j) - \alpha_i\alpha_j\mu^2} \tag{7}$$

and consequently the profit of the firm i is

$$\pi_i^*\,(\alpha_i,\alpha_j) = \frac{A^2\,(2+2c_j-\alpha_i\mu)\,(2(1+c_i+c_j+c_ic_j) - \mu\,(1+c_i)\,(2-\alpha_i) + \mu\alpha_j\,(1-\alpha_i)}{(4(1+c_i+c_j+c_ic_j) - \alpha_i\alpha_j\mu^2)^2} \tag{8}$$

for $i,j = 1,2 \quad i \neq j$.

If $\alpha_i\mu < 2\,(1+c_i)$ and $\alpha_j\mu \geq 2\,(1+c_j)$ then

$$q_i^*\,(\alpha_i,\alpha_j) = \frac{A}{2\,(1+c_i)} \ \text{ and } q_j^*\,(\alpha_i,\alpha_j) = 0, \ i,j = 1,2 \ , \ i \neq j \tag{9}$$

and firm i earns monopoly profit

$$\pi_i^*\,(\alpha_i,\alpha_j) = \frac{A^2}{4\,(1+c_i)}, \tag{10}$$

while firm j earns zero profit.

If $\alpha_i \mu \geq 2(1+c_i)$ for $i = 1, 2$ then there exist two pure strategy Nash equilibria in the second stage one in which firm 1 acts as a monopolist and firm 2 produces zero and vice versa.

Next we consider the reduced form game in stage 1 where each firm i maximizes its own profit $\pi_i^*(\alpha_i, \alpha_j)$ by choosing α_i. The manager of firm i maximizes

$$\max_{\alpha_i} q_i^*(\alpha_i, \alpha_j)(A - q_i^*(\alpha_i, \alpha_j) - \mu q_j^*(\alpha_i, \alpha_j) - c_i q_i^*(\alpha_i, \alpha_j)), \quad i, j = 1, 2, \ i \neq j. \tag{11}$$

If the interior Nash equilibrium in the second stage game given by (7) is substituted into (11) then the first order necessary condition for maximization yields

$$\left[4(1+c_i+c_j+c_ic_j) - 4\alpha_i(c_i+c_j+c_ic_j) - 4\mu\alpha_j(1+c_j) + 2\mu\alpha_i\alpha_j(1+c_j) + \mu^2\alpha_i\alpha_j\right] = 0 \tag{12}$$

It can be shown that the unique Nash equilibrium of the reduced form game in stage 1 is given by the unique solution to (12)

$$\alpha_i^C = \frac{2(1+c_j)(1-\mu+c_i)}{2(1+c_i+c_j+c_ic_j) - \mu(1+\mu+c_j)}, i, j = 1, 2, i \neq j, \tag{13}$$

(see appendix for the proof). In the subgame perfect Nash equilibrium of the two stage game the quantities chosen on the equilibrium path are given by

$$q_i^C = \frac{A\left[2(c_i+c_j+c_ic_j+1) - \mu(1+c_i+\mu)\right]}{4(1+c_i)(c_i+c_j+c_ic_j+1-\mu^2)}, i, j = 1, 2, i \neq j. \tag{14}$$

Observe that $0 < \alpha_i^C < 1$ which implies that each firm puts negative weight on rival's profit. Further,

$$c_1 < c_2 \Rightarrow \alpha_1^C < \alpha_2^C \tag{15}$$

which implies that $q_1^C > q_2^C$. Also, note that as μ increases both α_1^C and α_2^C increase. These observations can be summarized in terms of the following proposition.

Proposition 1 *The managerial incentive of the technologically efficient firm (which also acquires larger market share) is more sensitive to rival's profit (relative performance). As the degree of product differentiation* $\left(\frac{1}{\mu}\right)$ *decreases* *the equilibrium weights assigned by both firms to relative performance i.e. the absolute weights on rival's profit increase. With a decrease in the degree of product differentiation, the market share of the technologically efficient firm increases i.e., market concentration increases.*

It is worth comparing this strategic delegation model with the benchmark case where firms choose their quantities by maximizing own profit without delegating the task to the managers (henceforth, we refer this latter case as "non-delegation" model) i.e., $\alpha_1 = \alpha_2 = 1$. The unique Nash equilibrium of the second stage quantity game in the "non-delegation" model is given by

$$q_1^{ND} = \frac{A\left(2 + 2c_2 - \mu\right)}{4\left(1 + c_1 + c_2 + c_1 c_2\right) - \mu^2}, q_2^{ND} = \frac{A\left(2 + 2c_1 - \mu\right)}{4\left(1 + c_1 + c_2 + c_1 c_2\right) - \mu^2} \quad (16)$$

Let us define

$$\Delta = \frac{q_1^C}{q_1^C + q_2^C} - \frac{q_1^{ND}}{q_1^{ND} + q_2^{ND}} \quad (17)$$

where Δ reflects the difference caused in the market share of the relatively efficient firm through strategic delegation and this difference increases as product differentiation decreases. The following can be shown using (16).

Proposition 2 $\Delta > 0$ *and* Δ *is increasing in* μ *i.e. strategic delegation accentuates the asymmetry between firms in terms of their market shares* (*relative to "non- delegation"*) *and the magnitude by which this asymmetry is accentuated increases as the degree of product differentiation declines i.e., intensity of market competition increases.*

4 Price Competition

We now consider the two stage game where in the second stage firms compete in price competition. In particular we adopt a standard differentiated Bertrand model where demand faced by the firm i is given by

$$q_i\left(p_i, p_j\right) = A - p_i + \mu p_j, \quad i, j = 1, 2, i \neq j, \ \mu \leq 1 \quad (18)$$

We assume (unlike the previous section) that the firms produce under constant returns to scale and differ in their unit costs of production i.e., $c_1 < c_2$. Therefore, the cost function of the firm i is given by

$$C_i\left(q_i\right) = c_i q_i \ , \ i = 1, 2.$$

The reaction function of the second stage game is given by

$$
\begin{aligned}
p_i &= 0, \text{ if } \frac{A + c_i + \mu c_j}{2} + \frac{\alpha_i \mu}{2}(p_j - c_j) \leq 0, &(19)\\
&\geq A + \mu p_j, \text{ if } \frac{A + c_i + \mu c_j}{2} + \frac{\alpha_i \mu}{2}(p_j - c_j) \geq A + \mu p_j, &(20)\\
&= \frac{A + c_i + \mu c_j}{2} + \frac{\alpha_i \mu}{2}(p_j - c_j) \text{ otherwise for } i, j = 1, 2, \ i \neq j &(21)
\end{aligned}
$$

Note that if $\alpha_i \geq \frac{2}{\mu}$ for $i = 1, 2$ then the slope of the reaction function (on the right hand side of (21)) is greater than 1 so that the reaction functions may not intersect i.e. there may not exist any Nash equilibrium in pure strategies for the price competition game. Therefore, we restrict the space of contracts for each firm to:

$$\alpha_i < \frac{2}{\mu}, i = 1, 2. \tag{22}$$

It can be checked that both firms produce strictly positive output at the prices chosen in the Nash equilibrium of the second stage game provided, further, that

$$\alpha_i < \frac{2(A - c_i) + 2\mu(A + \mu c_i) + \mu^2 \alpha_j (A - c_i + \mu c_j)}{\mu\left[(A + \mu c_i - c_j) + \alpha_j \mu\left(A - c_i + \mu c_j\right)\right]}, \ i, j = 1, 2, \ i \neq j. \tag{23}$$

If (23) holds then the unique interior Nash equilibrium is given by

$$p_i^*\left(\alpha_i, \alpha_j\right) = \frac{2(A + c_i + \mu c_j) + \alpha_i \mu\left(A - c_j + \mu c_i\left(1 - \alpha_j\right)\right)}{4 - \mu^2 \alpha_i \alpha_j} \text{ for } \ i, j = 1, 2, i \neq j. \tag{24}$$

If the inequality in (23) is not satisfied for at least one firm, then in any Nash equilibrium one firm produces zero; in particular, if it is satisfied for firm i and not for firm j, then at every Nash equilibrium of the price subgame, firm j produces zero and firm i produces strictly positive quantity. As there are a continuum of equilibria when (23) is not satisfied, we refrain from characterizing the full set of equilibria in the price subgame for such cases; instead we can select any one of the equilibria and denote the prices by $p_i^*\left(\alpha_i, \alpha_j\right), i, j = 1, 2, i \neq j$.

We now consider the reduced form game in stage 1 where firms determine (α_1, α_2) subject to (22). In the reduced form game firm i maximizes

$$\max_{\alpha_i}(A - p_i^*\left(\alpha_i, \alpha_j\right) + \mu p_j^*\left(\alpha_i, \alpha_j\right))(p_i^*\left(\alpha_i, \alpha_j\right) - c_i).$$

It can be shown that[2] the unique *interior* Nash equilibrium of the reduced form game is given by

$$\alpha_i^B = \frac{2(A - c_j(1-\mu))}{A(2-\mu) + c_i\mu(1-\mu) - 2c_j(1-\mu)},\ i,j = 1,2, i \neq j. \tag{25}$$

The price and the quantity chosen on the equilibrium path in the second stage are

$$p_i^B = \frac{2(A + (1-\mu)c_i - \mu\left(A - (1-\mu)\,c_j\right)}{4(1-\mu)}, \tag{26}$$

$$q_i^B = \frac{2(A - c_i) + \mu\left(A + \mu c_i + c_j\right)}{4}. \tag{27}$$

From (26) and (27), it can be checked that $q_1^B > q_2^B$ and both firms earn strictly positive profit in equilibrium.

Note that $c_1 < c_2$ implies that

$$\alpha_1^B > \alpha_2^B > 1 \Rightarrow \left(\alpha_1^B - 1\right) > \left(\alpha_2^B - 1\right) > 0 \tag{28}$$

i.e. firm 1 assigns relatively greater positive weight on rival's profit in the the managerial incentive contract compared to her rival firm 2. To the extent that firms care about their own profit, the relatively efficient firm (firm 1) has a greater incentive to undercut and intensify price competition in the second stage and this intensification eventually affects its own profit adversely. This creates incentive for the more efficient firm to offer a contract to its manager that puts a greater positive weight on rival's profit (compared to the inefficient firm) so as to soften price competition. Lower the extent of product differentiation, higher the market incentive for price competition and therefore greater the relative incentive of the efficient firm (relative to the inefficient firm) to tie to the incentive of its manager to rival's profit so as to reduce his incentive to compete aggressively in prices in the second stage. In the limit i.e. as $\mu \to 1$, the managers' objective converge to joint profit maximization (perfect collusion).

[2] To see (25), note that in any interior equilibrium, (α_i, α_j) must satisfy (23) and, in that case, the second stage equilibrium prices are given by (24); using this for the above maximization problem and solving the first order conditions we obtain the expressions in (25). Note that $\left(\alpha_1^B, \alpha_2^B\right)$ satisfies (23). To check that $\left(\alpha_1^B, \alpha_2^B\right)$ is indeed an equilibrium of the reduced form game, we need to verify that neither firm can gain by unilaterally deviating to a choice of α_i such that (23) does not hold. This, however, follows immediately from the fact that, as indicated above, the deviating firm i will produce zero in the Nash equilibrium of the price subgame reached through such deviation and hence earn zero profit .

Proposition 3 *In equilibrium, the managerial compensation schemes of both firms assign positive weights to rival firm's profit and the sensitivity of compensation to rival's performance for both firms is decreasing in the extent of product differentiation. Managerial compensation in the relatively efficient firm (which has higher market share) is more sensitive to rival's profit (relative performance) than that in the inefficient firm. The difference between the sensitivity of managerial compensation to rival's profit in the two firms and the market share of the relatively inefficient firm is decreasing in the extent of product differentiation.*

The market share of each firm remains unaltered as compared to the benchmark case of "non-delegation" model i.e. $\alpha_1 = \alpha_2 = 1$.

5 Appendix

Claim 4 (α_1^c, α_2^c) *is the unique equilibrium.*
Proof. *First we establish that* (α_1^c, α_2^c) *is an equilibrium and in the next step we prove that it is indeed a unique equilibrium. For the former, fix* $\alpha_2 = \alpha_2^c$ *and allow firm* 1 *to alter* α_1 *from* α_1^c. *If firm* 1 *moves to any* $\alpha_1 < \frac{2(1+c_1)}{\mu}$ *both* α_1 *and* $\alpha_2 = \alpha_2^c$ *imply interior solutions of the second stage quantity game. Now given* $\alpha_2 = \alpha_2^c$ *from (12)* $\alpha_1 = \alpha_1^c$. *Therefore firm* 1 *will never reduce her* α_1 *below* $\frac{2(1+c_1)}{\mu}$. *On the other hand if* $\alpha_1 \geq \frac{2(1+c_1)}{\mu}$ *then from (7)* $\pi_1 = 0$. *Thus we can claim that* (α_1^c, α_2^c) *is an equilibrium.*

To establish the uniqueness, let us assume $\exists$ *an equilibrium* $(\widehat{\alpha}_1, \widehat{\alpha}_2)$ *in the first stage reduced form game such that:* (1)*if* $\widehat{\alpha}_1 \geq \frac{2(1+c_1)}{\mu}, \widehat{\alpha}_2 < \frac{2(1+c_2)}{\mu}$ *then in the second stage* $(q_1^* = 0, q_2^* = q_2^m = \frac{A}{2(1+c_1)})$ *from (9) and* $(\pi_1^* = 0, \pi_2^* = \pi_2^m = \frac{A^2}{4(1+c_2)})$ *from (10). Now observe that* $\frac{\partial \pi_1^*}{\partial \alpha_1}|_{(\alpha_1 = \frac{2(1+c_1)}{\mu}, \alpha_2 = \widehat{\alpha}_2)} < 0$ *i.e. at* $\alpha_1 = \left(\frac{2(1+c_1)}{\mu} - \epsilon\right)$ *firm* 1 *can earn strictly positive profit where* $\epsilon > 0$. (2) *If* $\widehat{\alpha}_1 < \frac{2(1+c_1)}{\mu}, \widehat{\alpha}_2 < \frac{2(1+c_2)}{\mu}$ *then in the second stage game*

$$\left(q_1^* = \frac{A(2(1+c_1) - \alpha_1\mu)}{4(1+c_1+c_2+c_1c_2) - \alpha_1\alpha_2\mu^2}, q_2^* = \frac{A(2(1+c_2) - \alpha_2\mu)}{4(1+c_1+c_2+c_1c_2) - \alpha_1\alpha_2\mu^2}\right)$$

from (7). From (12) we know that $\frac{\partial \pi_1^*}{\partial \alpha_1} = 0$ *at* $\alpha_1 = \alpha_1^c$. (3) *If* $\widehat{\alpha}_1 \geq \frac{2(1+c_1)}{\mu}, \widehat{\alpha}_2 \geq \frac{2(1+c_2)}{\mu}$ *then any firm can act as a monopolist and in that case same logical analysis as* (1) *will follow.*

Similar analysis can be done for $\widehat{\alpha}_2 \lesseqgtr \frac{2(1+c_2)}{\mu}$ *. This completes the proof.*
■

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