Quality Uncertainty and Time Inconsistency in a Durable Good Market

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Abstract

In a durable good monopoly where consumers cannot observe quality prior to purchase and product improvement occurs exogenously over time, we show that uncertainty in quality may resolve the time inconsistency problem (even for low levels of product improvement). Higher dispersion in quality creates greater demand for future product by increasing the incentive of buyers with inferior quality realizations to repurchase and this, in turn, reduces the incentive of the seller to cut future price. For various levels of product improvement, we characterize the range of quality uncertainty for which the market equilibrium is identical to one where the monopolist can credibly precommit to future prices. We also show that the presence of quality uncertainty can lead to no trading in the *primary* good market.

Keywords: durable goods, dynamic inconsistency, quality uncertainty.

JEL Classifications: L15, L12, D42, D81

*The author is grateful to Santanu Roy for helpful comments and suggestions.

1 Introduction

Markets for durable goods are characterized by a high degree of variance in realized quality. Further, many of the quality attributes cannot be observed by consumers prior to purchase even in the primary good market.[1] The degree of variation in realized quality affects the incentives of current buyers to sell in the used good market and repurchase a new good in the future. This, in turn, affects the incentive of sellers to reduce price of the new good in the future. Quality dispersion, therefore, affects the extent to which rational consumers expect prices to fall over time and the loss of market power arising from the incentive of buyers to wait for lower future prices that is described by the Coase conjecture (Coase (1972)).

As is well known, the Coase conjecture asserts that a durable good monopoly is subject to a time inconsistency problem in its pricing decision. When the monopolist is unable to credibly commit to a strategy of not lowering prices in the future, consumers refuse to pay a high price and instead prefer to wait for a lower future price. In equilibrium, the profit and market power are lower than in the situation where monopolist can credibly precommit. When quality is unobservable and uncertain, dispersion in quality creates market for the future product through inferior realizations of current quality; larger this dispersion, greater the incentive of buyers with inferior quality realizations to sell in the used good market and repurchase in the future new good market. This, we argue, reduces the incentive of the seller to cut future price and thus, mitigates partly or fully the Coasian loss of monopoly power due to time inconsistency.

An important factor in the decision to sell in the used good market and repurchase the new good in the future is the level of product improvement that occurs over time. Our analysis is therefore carried out in a framework that allows for (exogenous) product improvement. It is intuitive that even if there is no quality uncertainty, sufficiently high level of product improvement can resolve the time inconsistency problem in monopoly pricing by creating higher demand for improved future product and raising the incentive to charge a high future price. We show that if product improvement is not large enough (including the case of no product improvement), sufficient quality dispersion may resolve the time inconsistency problem. More generally, this paper analyzes how

[1] American Society for Quality (2006) reports the data for the "Differential Between Perceived Overall Quality and Customer Expectations for the Automobile Makers". The data shows that some nameplates have a large negative gap between the perceived overall quality and customer expectations (Mercedes-Benz, Volkswagen), whereas some others have a large positive gap (Hyundai, Saturn).

the interaction of product improvement and quality uncertainty affects monopoly pricing, market power and the time inconsistency problem. As the focus of this paper is on the effect of quality dispersion, we treat product improvement as exogenous and do *not* consider issues related to planned obsolescence, investment in product development, endogenous durability etc.[2]

Our analysis is carried out in a two period model of durable good monopoly. There are two types of risk neutral consumers: high and low valuation. The monopolist sells the existing product in the first period and an improved product in the second period. The realized quality of goods supplied is subject to exogenous uncertainty. In both periods, quality of a unit traded cannot be observed by the seller or by the buyer prior to purchase (information about quality is symmetric but incomplete in the primary market). After purchase, buyers observe the realized quality of their own good. In the second period, besides the primary market, there is also a competitive resale market where used goods may be sold by first period buyers; in this market, sellers have private information about the quality of the good they offer for sale.

The main results are as follows. First, we find that profit increases with an increase in product improvement. If product improvement is sufficiently large, even without precommitment, rational consumers can foresee that the seller will charge a relatively high price in the future and this reduces the discrepancy between the commitment and no-commitment outcomes. Thus, the Coase problem is resolved even though availability of improved future product tends to depress the first period price.[3]

Second, we find that when product improvement is small enough so that in the absence of dispersion in quality, the no-commitment profit is less than the commitment profit, an increase in quality uncertainty increases prices in both periods. For sufficiently high quality dispersion, the dynamic inconsistency (Coase) problem is resolved (i.e., the commitment and no-commitment equilibria are identical).

An increase in the spread of quality leads to two effects on the demand for new goods in the second period. The demand from first period

[2]In particular, we avoid issues related to time inconsistency in product improvement and focus exclusively on intertemporal pricing. For literature related to issues in dynamic product imrovement in Coasian framework see Waldman (1996) and Nahm (2004). Bulow (1986) shows that Coase time inconsistency problem can be mitigated by reducing durability.

[3]Notice that the mechanism which depresses the first period price is not through the usual Coasian dynamics but it is somewhat different. When Coase problem is resolved; that is, the monopolist commits to high future prices and regains its monopoly power, there is no remaining pressure on the first period price.

buyers who have inferior quality realizations increases but, at the same time, the potential demand from other first period buyers with better quality realizations falls. When product improvement is small and quality dispersion is high, to convince first period buyers with high quality realizations sell in the used good market and repurchase new good in the second period requires the seller to cut its second period price sharply. So the seller is better off ignoring such buyers and instead focusing exclusively on repurchases by low quality owners (who now have a high demand in the second period). This way, it is rational for the seller to charge a high price in the second period.

Note that as quality is not observed before purchase, the *ex ante* expected use-value of (risk-neutral) first period buyers is unaffected by dispersion of quality, but high dispersion creates a market segment of high-valuation consumers (with low quality realizations) that have a higher willingness to repurchase.

Third, we show that when product improvement is large, even though there is no time inconsistency, an increase in quality uncertainty may reduce profit. More interestingly, sufficiently high quality uncertainty can lead to no trading in the first period even if the commitment solution requires trading. When product improvement is large, the strategy to sell to all current owners (including those with high quality realization) in the second period becomes attractive to the seller. With sufficiently high dispersion in quality, the price cut required to be able to sell to all owners can be greater than the price charged for the existing product. In this case the seller shuts down the existing product market[4] in order to eliminate competition from high quality used goods;[5] that is, to avoid cannibalization of the sale of the improved product.

The existing literature on the durable good monopoly with exogenous product improvement focuses on the role of institutions such as buy-backs, upgrades and the joint sale of old and new generation products. See, among others, Fudenberg and Tirole (1998), Levinthal and Purohit (1989) and Lee and Lee (1998). Bond and Samuelson (1984) show that the Coase problem can be mitigated in the presence of exogenous depreciation (good is semi-durable). Our focus, however, is on the consequences of quality uncertainty on monopoly pricing and market power.

[4]When product improvement is sufficiently large, Levinthal and Purohit (1989) show that market is closed in the first period. The market closure in our model results from a sufficient increase in quality uncertainty, given a large product improvement. Levinthal and Purohit (1989) show this to be a consequence of product improvement.

[5]In the literature it has been shown that monopolist has sometimes incentive to eliminate the resale market. See Waldman (1997).

The literature on quality uncertainty has focused on adverse selection arising from asymmetric information (Akerlof (1970)). In our model, the used good market suffers from similar information problems. However, our focus is on the effects of quality uncertainty in the primary market where information is symmetric but incomplete. We show that quality uncertainty can distort, and even eliminate, trading in the primary market. More recently, Hendel and Lizzeri (1999) have emphasized the interaction between primary and secondary markets in reducing the problems emerging from adverse selection in the secondary market. However, this literature does not consider the effect of quality dispersion in the primary market.

The remainder of the paper is organized as follows. The next section presents the two period durable good monopoly model. Section 3 identifies the dynamic inconsistency problem with no quality uncertainty. The effect quality uncertainty on monopoly power and trading are examined in Sections 4 and 5. Section 6 concludes. The proofs are provided in the Appendix.

2 Model

Consider a two period durable good monopoly model. In period 1, the monopolist sells what we shall call the existing product. The existing product lasts for two periods (with no depreciation). A new product to be called the improved product is introduced to the market in period 2 with certainty. We assume that the existing product is phased out by the monopolist in period 2 - this could reflect limited production capacity. The existing product may however be traded in the resale market. In order to focus on the price differentials that emerge from dynamic market interactions rather than cost differences, we assume that both products are produced at an identical unit cost equal to zero.

The quality of units produced, denoted by s, is subject to exogenous variability arising from random shocks entering the production process. The quality of a new unit produced by a seller cannot be observed by the seller nor by the potential buyers prior to trade. The distribution of quality is common knowledge. That is, we have a problem of symmetric but incomplete information in the primary good market in each period. After trade occurs, buyers observe the realized quality of their purchase.

Quality dispersion is modelled by symmetric mean-preserving spreads as follows. The expected quality of the existing product is denoted by μ_1. There are two possible realizations of quality with equal probability:

low denoted by L and high denoted by H, where for some $t \in (0,1)$;

$$L = \mu_1(1-t)$$
$$H = \mu_1(1+t)$$

One can think of a unit of quality L as the defective product. An increase in t implies an increase in quality dispersion: the high quality unit gets better and the low quality unit gets worse.

The cost of development of the improved product introduced in period 2 is sunk prior to period 1 and therefore not considered explicitly in the model. The improved product has expected quality $\mu_2 = k\mu_1$, where $k > 1$ reflects the (exogenous) degree of product improvement which is common knowledge (since period 1). For our two period model, the actual statistical distribution of quality of the improved product is not important. Risk neutral consumers care only about the expected quality in the last period.

An owner always has the option of scrapping at zero cost if he so desires. In period 2, besides the primary market, there is also a competitive resale market where quality is not observable to buyers before purchase but known to sellers (existing owners).

There is a unit mass of risk neutral consumers who are born in period 1 and live for two periods. No new consumer enters the market in period 2. Each consumer owns at most one unit at a time. Consumers differ only in their valuations of quality. In particular, a type θ consumer who uses a unit of quality s derives utility flow θs *per period* of use. There are two different types of consumers: a portion $n \in (0,1)$ is of type $\overline{\theta}$ and a portion $1-n$ is of type $\underline{\theta}$, where $\underline{\theta} < \overline{\theta}$. The monopolist knows the distribution of valuations, but not the valuations of individual consumers. We assume that the discount rate is zero for both the monopolist and consumers.

In period 1, the monopolist sets the price for the existing product. Consumers then decide whether or not to buy. In period 2, resale price is determined competitively, and simultaneously, the monopolist sets the price for the improved product. Owners (first period buyers) decide whether to keep, scrap or sell their existing good (in the resale market). First period buyers who scrap or sell in the resale market as well as consumers who do not buy in the first period decide whether to buy and if so whether to buy in the secondary or the primary market.

In order to keep our analysis clean, we impose two additional restrictions on the parameters. First, we assume that there are more low-valuation consumers than high-valuation consumers, i.e.,

$$n < \frac{1}{2} \tag{1}$$

Second, we assume that

$$n\overline{\theta} > 2\underline{\theta} \tag{2}$$

i.e., there is sufficient heterogeneity in consumer valuations. Note that the standard Coasian time inconsistency problem becomes more severe as heterogeneity in consumer valuations increases.

3 Equilibrium with No Quality Uncertainty

As a benchmark, consider a durable good monopoly with no product improvement and no quality uncertainty, i.e., $k = 1$, $t = 0$. In both periods, the same product is sold by the monopolist. As Coase conjectures, when the monopolist is not able to precommit to future price, the market equilibrium is one where high-valuation consumers buy in period 1 at price $p_1 = (\overline{\theta} + \underline{\theta})\mu_1$, and in period 2, the monopolist reduces the price and sells to low-valuation consumers at the maximum price they are willing to pay, i.e., $p_2 = \underline{\theta}\mu_1$. Observe that anticipating the price cut in period 2, high-valuation consumers are willing to pay only $\overline{\theta}\mu_1 + p_2$ in period 1. Also observe that there is no trading in the resale market. The profit of the monopolist is $\mu_1(\overline{\theta}n + \underline{\theta})$. If the monopolist could precommit to the price in period 2, it is optimal for him to set p_2 high enough so that there is no trading in period 2 and charge $p_1 = 2\overline{\theta}\mu_1$ which would be acceptable to all high-valuation consumers. These prices give rise to the profit of $2\overline{\theta}n\mu_1$ that is higher than the no-commitment profit. This illustrates the basic dynamic inconsistency problem in pricing faced by the durable good monopolist - consumers anticipate future price reduction and tend to wait for the lower future price, and this reduces the ability of the seller to charge a high price now, leading to a consequent loss of monopoly power.

The next two propositions show the effect of product improvement ($k > 1$) on the Coase time inconsistency problem while continuing to assume zero dispersion in quality ($t = 0$). First, we consider the market equilibrium with no commitment. With product improvement, in period 2 there is an active resale market where owners sell their used goods at price p_u.

Proposition 1 *Assume that $t = 0$, i.e., there is no quality dispersion. The no-commitment market equilibrium is as follows. Only high-valuation consumers buy in period 1 (at price $p_1 = (\overline{\theta} + \underline{\theta})\mu_1$) and in period 2, these consumers sell in the resale market at price $p_u = \underline{\theta}\mu_1$ (to low-valuation consumers) and repurchase the improved product.*

(i) *If $k < \frac{\overline{\theta}-\underline{\theta}}{\overline{\theta}-\underline{\theta}(\frac{1-n}{n})}$, the improved product in period 2 is sold at price $p_2 = \underline{\theta}k\mu_1$; low-valuation consumers are indifferent between buying the*

used good and the improved product in period 2.
(ii) *If* $k \geq \frac{\overline{\theta}-\underline{\theta}}{\overline{\theta}-\underline{\theta}(\frac{1-n}{n})}$, *the improved product in period 2 is sold at price* $p_2 = [\overline{\theta}(k-1)+\underline{\theta}]\mu_1$; *low-valuation consumers are indifferent between buying the used good in period 2 and not buying at all.*

The main difference caused by product improvement to the no-commitment market equilibrium is that high-valuation consumers sell their good in the resale market and repurchase the improved product in period 2, and this allows the monopolist to charge a high price in period 2. When product improvement is sufficiently large, i.e., $k \geq \frac{\overline{\theta}-\underline{\theta}}{\overline{\theta}-\underline{\theta}(\frac{1-n}{n})}$, the difference between the willingness to pay of high and low valuation consumers for the improved product is also large so that the monopolist finds it profitable to ignore low-valuation consumers and only make repeat sales to high-valuation consumers in period 2 at a high price. Product improvement eliminates the tendency to wait for the future price cut. Nevertheless, with product improvement, consumers tend to wait for the better future product and this tends to reduce the willingness to pay and thus the price in period 1.

The next proposition outlines the precommitment monopoly solution.

Proposition 2 *Assume that* $t = 0$, *i.e., there is no quality dispersion. The market equilibrium when the monopolist can credibly precommit to future price is as follows.*
(i) *If* $k < 2(1-\frac{\underline{\theta}}{\overline{\theta}})$, *commitment equilibrium is identical to the commitment equilibrium with no product improvement. Only high-valuation consumers buy the existing product (at price* $p_1 = 2\overline{\theta}\mu_1$*). There is no trading in period 2.*
(ii) *If* $k \geq 2(1-\frac{\underline{\theta}}{\overline{\theta}})$, *commitment equilibrium is identical to (ii) of Proposition 1.*

When product improvement is relatively small so that $k < 2(1-\frac{\underline{\theta}}{\overline{\theta}})$, the monopolist continues to find it optimal to follow the same strategy as he does when there is no product improvement, i.e., precommit to such a high p_2 so that there is no trading in period 2 and sell to high-valuation consumers in period 1 at their maximum willingness to pay.

When product improvement is sufficiently high so that $k \geq 2(1-\frac{\underline{\theta}}{\overline{\theta}})$, high-valuation consumers have such high willingness to pay for the improved product that it makes sense for the monopolist to make repeat sales to high-valuation consumers in period 2 (without reducing price in period 2 significantly).

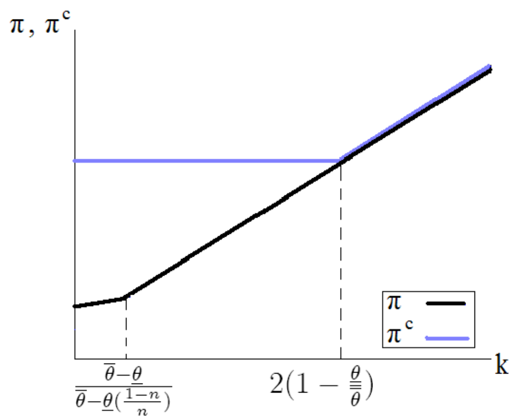

Figure 1: Commitment (π^c) and no-commitment (π) profit functions with no quality uncertainty

As there is sufficient heterogeneity in valuations by assumption (2), it is easy to see that

$$2(1-\frac{\underline{\theta}}{\overline{\theta}}) \geq \frac{\overline{\theta}-\underline{\theta}}{\overline{\theta}-\underline{\theta}(\frac{1-n}{n})}$$

Therefore, the next Corollary follows from Propositions 1 and 2.

Corollary *Suppose there is no quality uncertainty. There is no time inconsistency problem if and only if* $k \geq 2(1-\frac{\underline{\theta}}{\overline{\theta}})$.

As illustrated in Figure 1, an increase in product improvement diminishes the gap between the commitment (π^c) and no-commitment (π) profits. Product improvement mitigates the time inconsistency problem and resolves it completely when $k \geq 2(1-\frac{\underline{\theta}}{\overline{\theta}})$. Even though the downward pressure on p_1 caused by product improvement remains, $\pi = \pi^c$ because repeat sales also depress the commitment price in period 1.

4 Quality Uncertainty and Time Inconsistency

We now introduce quality uncertainty. Besides valuations, owners are also differentiated according to the realizations of quality. A type $\theta \in \{\overline{\theta}, \underline{\theta}\}$ consumer who buys an s quality good in period 1, is said to be of θ_s in period 2, where $s \in \{L, H\}$. A type $\theta \in \{\overline{\theta}, \underline{\theta}\}$ consumer who does not buy in period 1 is said to be of type θ_0 in period 2. In period 2, the market therefore may potentially consist of different types of consumers: $\overline{\theta}_L, \overline{\theta}_H, \overline{\theta}_0, \underline{\theta}_H, \underline{\theta}_L, \underline{\theta}_0$ depending on the market outcome in period 1. For example if only high-valuation (type $\overline{\theta}$) consumers buy in period 1, in period 2 there are three different types of consumers; namely, types $\overline{\theta}_L$, $\overline{\theta}_H$ and $\underline{\theta}_0$.

A type $\theta \in \{\overline{\theta}, \underline{\theta}\}$ consumer buys the existing product if his willingness to pay is at least as much as p_1 :

$$\theta\mu_1 + \frac{v_2(\theta_L) + v_2(\theta_H)}{2} - v_2(\theta_0) \geq p_1 \qquad (IC_1(\theta))$$

where $v_2(\theta_L)$, $v_2(\theta_H)$ and $v_2(\theta_0)$ represent the continuation values[6] of types θ_L, θ_H and θ_0, respectively.

Quality uncertainty creates used goods of two different levels of quality: L and H. In period 2, owners decide whether to keep or dispose of their used goods (either by selling in the resale market or by scrapping it), and in the latter case whether to buy a new good, buy a used good or not buy at all. Similarly, type θ_0 consumers decide whether to buy a new good, buy a used good or not buy at all. The continuation values of types θ_0, θ_L and θ_H are as follows:

$$\begin{aligned} v_2(\theta_0) &= \max\{\theta\mu_2 - p_2, \theta\mu_u - p_u, 0\} \\ v_2(\theta_L) &= \max\{\theta\mu_2 - p_2, \theta\mu_u - p_u, \theta L - p_u, 0\} + p_u \\ v_2(\theta_H) &= \max\{\theta\mu_2 - p_2, \theta\mu_u - p_u, \theta H - p_u, 0\} + p_u \end{aligned}$$

where μ_u represents the expected quality of the goods which are offered for sale in the resale market.

In period 2, consumers buy the improved product if their willingness to pay for the new unit is at least as much as p_2. The willingness to pay for the improved product differs depending on the valuation and ownership of the consumers. For $\theta \in \{\overline{\theta}, \underline{\theta}\}$, type θ_0, θ_L and θ_H consumers buy the improved product if the following incentive constraints are respectively satisfied:

$$\theta\mu_2 - \max\{\theta\mu_u - p_u, 0\} \geq p_2 \qquad (IC_2(\theta_0))$$

$$\theta\mu_2 - \max\{\theta L - p_u, \theta\mu_u - p_u, 0\} \geq p_2 \qquad (IC_2(\theta_L))$$

$$\theta\mu_2 - \max\{\theta H - p_u, \theta\mu_u - p_u, 0\} \geq p_2 \qquad (IC_2(\theta_H))$$

Resale market has two effects on the willingness to pay for the improved product. On the one hand, resale market creates competition which reduces the willingness to pay for the improved product. On the other hand, as each consumer owns at most one unit, repeat buyers are also used good suppliers. Thus the resale market provides the adequate incentives to repeat buyers to repurchase by increasing their willingness

[6]The continuation value of a consumer is the net surplus he gets from that point onwards.

to pay for the improved product. Therefore, the equilibria in the primary and the resale markets are determined interactively.

Considering these dynamics between primary and resale markets, the demand schedule for the improved product in period 2 is derived as a step function. The demand schedule is shown in Table 1, and further explanations are given in the Appendix. The following lemma is useful in understanding the demand schedule for the improved product.

Lemma *In period 1, low-valuation consumers buy the existing product only if high-valuation consumers buy.*

The lemma shows that a market outcome at which only low-valuation consumers buy the existing product is not possible. Therefore, there are three possibilities in period 1: no trading, purchase only by high-valuation consumers, or purchase by all consumers. In each case, the market consists of different types of consumers in period 2. That is, the demand schedule for the improved product depends on the market outcome in period 1. The demand schedule differs also depending on the degree of product improvement. Notice that for product improvement $k < 1 + t$, H quality owners already own a product of higher quality than the improved product; $\mu_2 < H$.

In what follows, we use the following categorization of the degree of product improvement.

Definition *Product improvement is said to be*

high	*if*	$4 \leq k$,
intermediate	*if*	$2(1 - \frac{\underline{\theta}}{\overline{\theta}}) \leq k < 4$, *and*
low	*if*	$k < 2(1 - \frac{\underline{\theta}}{\overline{\theta}})$.

Note that in the analysis with no quality uncertainty, we have seen that if product improvement is low, there is a time inconsistency problem. In the next three subsections we will study the effect of change in quality uncertainty for each of these categories of product improvement.

4.1 High Product Improvement

As discussed, quality uncertainty creates dispersion in the willingness to repurchase. The required price cut which induces consumers with the same valuation to repurchase differs depending on the realizations of quality. However, with high product improvement, the quality of the improved product is so high that in period 2, both the monopolist and consumers behave almost as if a completely different product is on the market for the first time. Irrespective of the market outcome in period 1, the monopolist limits output only to the purchases of high-valuation consumers as in a static monopoly.

TABLE 1 - The Demand Schedule for the Improved Product

		$\mathbf{p_2}$		$\mathbf{q_2}$	
		If there is no trading in period 1: $\mathbf{q_1 = 0}$			
(1)	$\overline{\theta}\mu_2$	$< p_2$		0	
(2)	$\underline{\theta}\mu_2$	$< p_2 \leq$	$\overline{\theta}\mu_2$	n	
(3)		$p_2 \leq$	$\underline{\theta}\mu_2$	1	
		If only type $\overline{\theta}$ consumers buy in period 1: $\mathbf{q_1 = n}$			
▶ **If** $k < 1 + t\frac{\overline{\theta}}{\overline{\theta}-\underline{\theta}}$					
(4)	$\overline{\theta}(\mu_2 - L) + \underline{\theta}L$	$< p_2$		0	
(5)	$\underline{\theta}\mu_2$	$< p_2 \leq$	$\overline{\theta}(\mu_2 - L) + \underline{\theta}L$	$\frac{n}{2}$	
(6)	$\underline{\theta}(\mu_2 - L)$	$< p_2 \leq$	$\underline{\theta}\mu_2$	$1 - n$	
(7)	$\overline{\theta}(\mu_2 - H)$	$< p_2 \leq$	$\underline{\theta}(\mu_2 - L)$	$1 - \frac{n}{2}$	
(8)		$p_2 \leq$	$\overline{\theta}(\mu_2 - H)$	1	if $k \geq 1 + t$; else $1 - \frac{n}{2}$
▶ **If** $k \geq 1 + t\frac{\overline{\theta}}{\overline{\theta}-\underline{\theta}}$					
(9)	$\overline{\theta}(\mu_2 - L) + \underline{\theta}L$	$< p_2$		0	
(10)	$\overline{\theta}(\mu_2 - H) + \underline{\theta}\mu_1$	$< p_2 \leq$	$\overline{\theta}(\mu_2 - L) + \underline{\theta}L$	$\frac{n}{2}$	
(11)	$\underline{\theta}\mu_2$	$< p_2 \leq$	$\overline{\theta}(\mu_2 - H) + \underline{\theta}\mu_1$	n	
(12)	$\underline{\theta}(\mu_2 - \mu_1)$	$< p_2 \leq$	$\underline{\theta}\mu_2$	$1 - n$	
(13)		$p_2 \leq$	$\underline{\theta}(\mu_2 - \mu_1)$	1	
		If both types buy in period 1: $\mathbf{q_1 = 1}$			
▶ **If** $k < 1 + t\frac{\overline{\theta}+\underline{\theta}(\frac{1-n}{1+n})}{\overline{\theta}-\underline{\theta}}$					
(14)	$\overline{\theta}(\mu_2 - L)$	$< p_2$		0	
(15)	$\underline{\theta}(\mu_2 - L)$	$< p_2 \leq$	$\overline{\theta}(\mu_2 - L)$	$\frac{n}{2}$	
(16)	$\overline{\theta}(\mu_2 - H)$	$< p_2 \leq$	$\underline{\theta}(\mu_2 - L)$	$\frac{1}{2}$	
(17)	$\underline{\theta}(\mu_2 - H)$	$< p_2 \leq$	$\overline{\theta}(\mu_2 - H)$	$\frac{1+n}{2}$	if $k \geq 1 + t$; else $\frac{1}{2}$
(18)		$p_2 \leq$	$\underline{\theta}(\mu_2 - H)$	1	if $k \geq 1 + t$; else $\frac{1}{2}$
▶ **If** $1 + t\frac{\overline{\theta}+\underline{\theta}(\frac{1-n}{1+n})}{\overline{\theta}-\underline{\theta}} \leq k < 1 + t\frac{\overline{\theta}+\underline{\theta}}{\overline{\theta}-\underline{\theta}}$					
(19)	$\overline{\theta}(\mu_2 - L)$	$< p_2$		0	
(20)	$\underline{\theta}(\mu_2 - L)$	$< p_2 \leq$	$\overline{\theta}(\mu_2 - L)$	$\frac{n}{2}$	
(21)	$\overline{\theta}(\mu_2 - H)$	$< p_2 \leq$	$\underline{\theta}(\mu_2 - L)$	$\frac{1}{2}$	
(22)	$\underline{\theta}(\mu_2 - \frac{L+nH}{1+n})$	$< p_2 \leq$	$\overline{\theta}(\mu_2 - H)$	n	
(23)	$\underline{\theta}(\mu_2 - H)$	$< p_2 \leq$	$\underline{\theta}(\mu_2 - \frac{L+nH}{1+n})$	$\frac{1+n}{2}$	
(24)		$p_2 \leq$	$\underline{\theta}(\mu_2 - H)$	1	
▶ **If** $k \geq 1 + t\frac{\overline{\theta}+\underline{\theta}}{\overline{\theta}-\underline{\theta}}$					
(25)	$\overline{\theta}(\mu_2 - L)$	$< p_2$		0	
(26)	$\overline{\theta}(\mu_2 - H)$	$< p_2 \leq$	$\overline{\theta}(\mu_2 - L)$	$\frac{n}{2}$	
(27)	$\underline{\theta}(\mu_2 - \frac{L+nH}{1+n})$	$< p_2 \leq$	$\overline{\theta}(\mu_2 - H)$	n	
(28)	$\underline{\theta}(\mu_2 - H)$	$< p_2 \leq$	$\underline{\theta}(\mu_2 - \frac{L+nH}{1+n})$	$\frac{1+n}{2}$	
(29)		$p_2 \leq$	$\underline{\theta}(\mu_2 - H)$	1	

Proposition 3 *Suppose product improvement is high: $k \geq 4$. There is no time inconsistency problem.*

With high product improvement, profit from sales to only high-valuation consumers is so large that even with the ability to commit, the monopolist commits to a high p_2 at which only high-valuation consumers buy. Commitment and no-commitment equilibria are identical as in the analysis with no quality uncertainty.

4.2 Low Product Improvement

Now we turn to the opposite case in which product improvement is low: $k < 2(1 - \frac{\underline{\theta}}{\overline{\theta}})$. In period 2, as product improvement is low, the willingness to purchase is low in general. An increase in quality uncertainty reduces the willingness to purchase of H quality owners further. As quality uncertainty increases, H quality used goods become a stronger competitor for the improved product. It is difficult to convince H quality owners to replace their existing good. On the other hand, an increase in quality uncertainty increases the willingness to purchase of L quality owners. This, in turn, increases the price cut required to sell to consumers other than L quality owners. Therefore, when quality uncertainty is sufficiently high, it is profitable to ignore other types and limit output only to the repurchases by L quality owners.

Note that L quality owners in period 2 are buyers with such a high valuation that they purchase in period 1 rather than waiting, but realized exposed that they happen to have purchased a defective product. Therefore, they have high willingness to pay for the improved product. The monopolist, therefore, finds it optimal to choose a high p_2.

With no quality uncertainty, we have shown that all high-valuation consumers repurchase. Therefore, there is a downward pressure on p_1. With quality uncertainty, there is a positive probability of buying an H quality product and using it for two periods. This possibility increases the willingness to pay for the existing product in period 1.[7] That is, quality uncertainty mitigates the pressure on p_1 for low levels of product improvement.

The next proposition shows that if quality uncertainty is sufficiently large, time inconsistency problem disappears and market power of the monopolist is as high as under full commitment.

Proposition 4 *Suppose product improvement is low: $k < 2(1 - \frac{\underline{\theta}}{\overline{\theta}})$. There is no time inconsistency problem if and only if $t \geq 1 - \frac{k \min\left\{\frac{\overline{\theta}}{3}, \overline{\theta} - 2\underline{\theta}(\frac{1-n}{n})\right\}}{\overline{\theta} - \underline{\theta}}$.*

[7] Since quality is not observed before purchase and consumers are risk neutral, willingness to pay for the existing product is not affected by the realizations of quality.

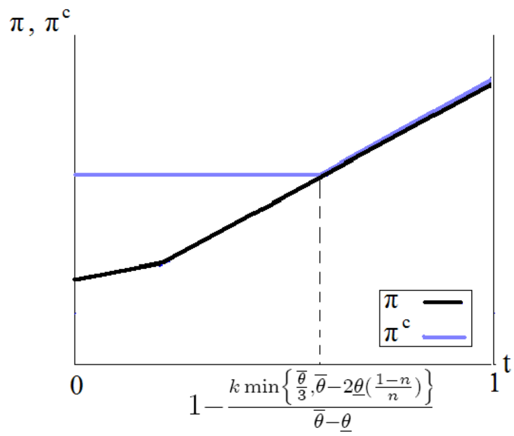

Figure 2: Commitment (π^c) and no-commitment (π) profit functions with low product improvement

In period 1, consumers pay for the existing product with expected quality μ_1, but in period 2 the marginal consumer,[8] i.e., type $\overline{\theta}_L$ requires a price cut as a defective product owner. Therefore, if quality uncertainty is sufficiently high, by limiting the output of the improved product to only the repurchases by defective product owners, the monopolist is able to charge a higher price in both periods and resolve the time inconsistency problem (see Figure 2). Even with no product improvement, Coase time inconsistency problem is resolved at sufficiently high levels of quality uncertainty.

4.3 Intermediate Product Improvement

Previous analysis suggests that quality uncertainty helps to resolve time inconsistency problem. The following proposition shows that increase in quality uncertainty does not necessarily reduce the degree of time inconsistency for intermediate product improvement. In fact, time inconsistency problem can arise for higher levels of quality uncertainty even if it does not arise for low levels of quality uncertainty.

Proposition 5 *Suppose* $2(2-\frac{\underline{\theta}}{\overline{\theta}}) \leq k < 4$. *There is no time inconsistency problem if and only if* $t \leq 1-\frac{\overline{\theta}(4-k)}{3(\overline{\theta}-\underline{\theta})}$.

Just like the previous analysis, for intermediate product improvement, maximum profit (π) could be attained by limiting the period 2 output to the repurchases by defective product owners. However, in

[8] The marginal consumer is the consumer with the lowest willingness to pay among the consumers who purchase.

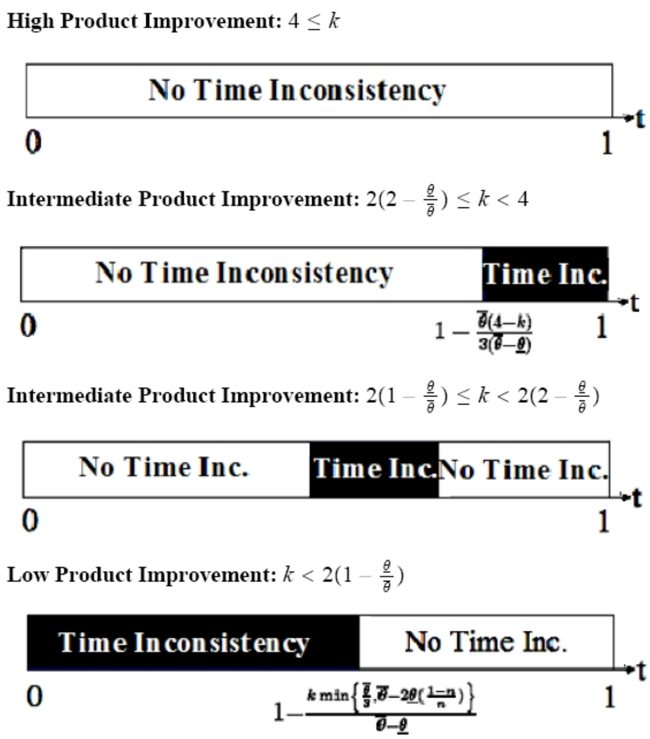

Figure 3: Time Inconsistency Problem

period 2, for higher levels of product improvement, the loss in period 2 profit (π_2) from limiting output outweighs the gain from the ability to charge a higher p_2. So, with no ability to commit, as product improvement increases, the range of quality uncertainty where the output is limited shrinks, and it disappears if $2(2 - \frac{\underline{\theta}}{\bar{\theta}}) \leq k$ (see Figure 3).

For lower levels of product improvement within the intermediate range, i.e., $2(1 - \frac{\underline{\theta}}{\bar{\theta}}) \leq k < 2(2 - \frac{\underline{\theta}}{\bar{\theta}})$, one can show that both sufficiently low[9] and sufficiently high levels of quality uncertainty resolve the time inconsistency problem, but time inconsistency problem reemerges if t is intermediate.

5 Quality Uncertainty and Trading

In the analysis with no quality uncertainty we have seen that trading takes place in period 1, irrespective of the ability to commit. We now show that if both product improvement and quality uncertainty are sufficiently large, trading in period 1 breaks down even if the monopolist has ability to commit.

[9]For $t = 0$, Corollary states that there is no time inconsistency problem.

Proposition 6 *Suppose* $t \geq 2\frac{\underline{\theta}}{\overline{\theta}}$ *and* $k \geq \max\left\{2, \frac{(1+3t)\overline{\theta}+3(1-t)\underline{\theta}}{\overline{\theta}}\right\}$. *There is no trading in period 1 even with the ability to commit.*

If product improvement is sufficiently large, i.e., $k \geq \frac{(1+3t)\overline{\theta}+3(1-t)\underline{\theta}}{\overline{\theta}}$, irrespective of the market outcome in period 1, in period 2 the monopolist sells to all high-valuation consumers. Note that if there is no trading in period 1, in period 2, the marginal consumer is of type $\overline{\theta}_0$, or else if the monopolist sells in period 1, it is of type $\overline{\theta}_H$. The maximum price the monopolist could charge is determined by the maximum willingness of the marginal consumer to pay for the improved product. If there is no trading in period 1, i.e., $(q_1, q_1) = (0, n)$, it is given in the demand schedule as $p_2(0, n) = \overline{\theta}\mu_2$; conversely if the monopolist sells to high-valuation consumers in period 1, i.e., $(q_1, q_1) = (n, n)$, it is given as $p_2(n, n) = \overline{\theta}(\mu_2 - H) + \underline{\theta}\mu_1$. If the monopolist sells the existing product, he needs to cut the price to convince the marginal consumer (type $\overline{\theta}_H$) to repurchase. The required price cut $p_2(0, n) - p_2(n, n)$ increases with an increase in quality uncertainty. In particular, when $t \geq 2\frac{\underline{\theta}}{\overline{\theta}}$, the required price cut is even greater than the price charged for the existing product: $p_2(0, n) - p_2(n, n) \geq p_1(n, n)$.[10] Since quality can not be observed before purchase, existing product buyers pay for a good with expected quality μ_1, but in period 2, the marginal consumer requires a price cut as an H quality owner.

Therefore, the monopolist is better off by not trading in period 1 and then selling to first time buyers (type $\overline{\theta}_0$) in period 2 rather than competing with the H quality used goods. The existing product market is shut down in order to eliminate the H quality used goods. The monopolist leapfrogs to the improved product and the market structure is a static monopoly. This is depicted in Figure 4, for the parameter values $\overline{\theta} = 1$, $\underline{\theta} = 0.18$, $n = 0.4$ and $\mu_1 = 1$.

When product improvement is lower, the monopolist with ability to commit finds it profit maximizing to sell in period 1. Most interestingly at these lower levels of product improvement, inability to commit leads to break down of trading.

Proposition 7 *Suppose* $t \geq 2\frac{\underline{\theta}}{\overline{\theta}}$ *and* $\max\left\{2, \frac{(1+3t)\overline{\theta}-(1+t)\underline{\theta}}{\overline{\theta}}\right\} \leq k < \frac{(1+3t)\overline{\theta}+3(1-t)\underline{\theta}}{\overline{\theta}}$. *There is no trading in period 1 as a consequence of time inconsistency.*

For lower levels of product improvement, i.e., $\max\left\{2, \frac{(1+3t)\overline{\theta}-(1+t)\underline{\theta}}{\overline{\theta}}\right\} \leq k < \frac{(1+3t)\overline{\theta}+3(1-t)\underline{\theta}}{\overline{\theta}}$, no trading takes place even if the commitment solution requires trading. With ability to commit, it is optimal for the monopolist to sell to high-valuation consumers in period 1 and then limit

[10] Given $p_2(n, n)$, $p_1(n, n)$ is found in the Appendix (Table 2) as $p_1(n, n) = (\overline{\theta}+\underline{\theta})\mu_1$.

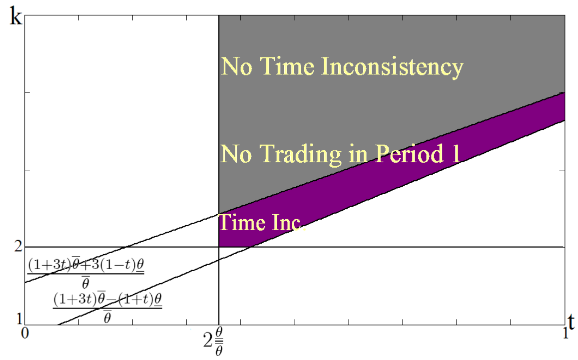

Figure 4: No Trading in Period 1

period 2 output to the repurchases by defective product owners rather than no trading in period 1 ($\pi(n, \frac{n}{2}) > \pi(0, n)$). But having sold to high-valuation consumers in period 1, in period 2, the monopolist finds it profitable not to limit output ($\pi_2(n, n) > \pi_2(n, \frac{n}{2})$). Instead, he sells to all owners. However, profit from no trading is greater than the profit from selling to all owners ($\pi(0, n) > \pi(n, n)$). The monopolist shuts down the existing product market and leapfrogs to the improved product as he can not credibly commit to a high p_2 at which only defective product owners buy. The time inconsistency problem leads to no trading in period 1 (see Figure 4).

6 Conclusion

In this paper, we characterized the exact degree of product improvement and quality uncertainty at which the time inconsistency problem is resolved. We also analyzed the effect of the interaction between quality uncertainty and product improvement on trading.

There are several directions in which the analysis in this paper could be extended. One could endogenize the degree of product improvement.[11] The degree of quality uncertainty could also be endogenized by introducing a quality control process. In this set up, the questions of interest are how much to improve the existing product and how much quality uncertainty to allow. The answers to these questions depend on the costs of product improvement and quality control. Yet, even without incorporating the costs, our analysis sheds some light on the incentive for doing quality control.

As quality uncertainty increases, H quality used goods become a better product. Competition with the H quality used goods cannibalizes

[11] Lee and Lee (1998) investigates the choice of R&D with no quality uncertainty.

sales of the improved product. With high product improvement, this is more detrimental to the monopolist since increases in quality uncertainty lead to no trading in period 1, reducing the profit. So the monopolist may like to do a strict quality control to reduce quality uncertainty in order to be able to sell the existing product.

On the other hand, with low product improvement, we have already seen that the profit increases with increases in quality uncertainty. Therefore, the monopolist may not have incentive to invest in quality control even at a cost of zero.

Appendix

We first solve for some preliminary results which are used in the proofs of the propositions. When $t = 0$, the analysis with quality uncertainty mirrors that with no quality uncertainty. To avoid repetition, the preliminary results are given for the general case at which $t \in [0, 1]$.

Proof of Lemma. Let $s \in \{L, H\}$ and $s' = \{L, H\} \setminus s$. Denote $v_2(\theta_0)$ and $v_2(\theta_s)$ for consumers with valuations $\overline{\theta}$ and $\underline{\theta}$ as $v_2(\overline{\theta}_0)$, $v_2(\overline{\theta}_s)$ and $v_2(\underline{\theta}_0)$, $v_2(\underline{\theta}_s)$, respectively. A type $\overline{\theta}_s$ consumer can dispose and act like a type $\overline{\theta}_0$, so

$$v_2(\overline{\theta}_s) \geq p_u + v_2(\overline{\theta}_0) \tag{3}$$

Denote $IC_1(\theta)$ for types $\overline{\theta}$ and $\underline{\theta}$ as $IC_1(\overline{\theta})$ and $IC_1(\underline{\theta})$, respectively. Suppose type $\underline{\theta}$ consumers buy the existing product; that is, $IC_1(\underline{\theta})$ holds. Consumers with valuation $\underline{\theta}$ can be divided into two different types: $\underline{\theta}_s$ and $\underline{\theta}_{s'}$. In period 2, either both types dispose, both keep or one type keeps while the other disposes. For each case, we are going to show that $IC_1(\overline{\theta})$ is also satisfied.

If both types dispose in period 2, $\frac{v_2(\underline{\theta}_L)+v_2(\underline{\theta}_H)}{2} = p_u + v_2(\underline{\theta}_0)$. $IC_1(\underline{\theta})$ reduces to $\underline{\theta}\mu_1 + p_u \geq p_1$. Given (3), at this price range $IC_1(\overline{\theta})$ holds. If both types keep in period 2, $\frac{v_2(\underline{\theta}_L)+v_2(\underline{\theta}_H)}{2} = \underline{\theta}\mu_1$. $IC_1(\underline{\theta})$ reduces to $2\underline{\theta}\mu_1 - v_2(\underline{\theta}_0) \geq p_1$. Given (3) and (2), at this price range $IC_1(\overline{\theta})$ holds. Finally, if one of the types ($\underline{\theta}_{s'}$) disposes, i.e., $v_2(\underline{\theta}_0) \geq \underline{\theta}s' - p_u$ and the other one ($\underline{\theta}_s$) keeps, $\frac{v_2(\underline{\theta}_L)+v_2(\underline{\theta}_H)}{2} = \frac{\underline{\theta}s+p_u+v_2(\underline{\theta}_0)}{2}$. $IC_1(\underline{\theta})$ reduces to $\underline{\theta}\mu_1 + \frac{\underline{\theta}s-v_2(\underline{\theta}_0)+p_u}{2} \geq p_1$. Given $v_2(\underline{\theta}_0) \geq \underline{\theta}s' - p_u$, (3) and (2), at this price range $IC_1(\overline{\theta})$ holds. ∎

The Demand Schedule for the Improved Product

To derive the demand schedule for the improved product, we first find out the type of consumers who are willing to pay the highest price for the improved product, given that no other type of consumers buy. Then, given that the first type of consumers buy, we find the type which would be willing to pay the second highest price, and so on. Since the demand of each type depends on the market outcome in the resale market, i.e., the purchases of other types, we check in each iteration whether the earlier types are still willing to purchase, together with the latter types.

Below for $q_1 = n$ and $k < 1+t\frac{\underline{\theta}}{\overline{\theta}-\underline{\theta}}$ ((4) to (8) in Table 1), we show that given p_2, there is a p_u and μ_u at which demand for the new product is q_2. The demand schedule for the other ranges can be validated similarly. Denote $IC_2(\theta_0)$, $IC_2(\theta_L)$ and $IC_2(\theta_H)$ for consumers with valuations $\overline{\theta}$ and $\underline{\theta}$ as $IC_2(\overline{\theta}_0)$, $IC_2(\overline{\theta}_L)$, $IC_2(\overline{\theta}_H)$ and $IC_2(\underline{\theta}_0)$, $IC_2(\underline{\theta}_L)$, $IC_2(\underline{\theta}_H)$, respectively. If $q_1 = n$, consumers can be divided into three different types: $\frac{n}{2}$ of them is of type $\overline{\theta}_L$, $\frac{n}{2}$ of them is of type $\overline{\theta}_H$ and $1-n$ of them is of type $\underline{\theta}_0$.

(4) If $\overline{\theta}(\mu_2 - L) + \underline{\theta}L < p_2$, $p_u = \underline{\theta}L$ and $\mu_u = L$. Given p_2, p_u and μ_u, one can show that $IC_2(\overline{\theta}_L)$, $IC_2(\overline{\theta}_H)$ and $IC_2(\underline{\theta}_0)$ does not hold. Owners keep. Since there is no trade in the resale market μ_u is not defined. We suppose in this case $\mu_u = L$. Nobody buys new: $q_2 = 0$.

(5) If $\underline{\theta}\mu_2 < p_2 \leq \overline{\theta}(\mu_2 - L) + \underline{\theta}L$, $p_u = \underline{\theta}L$ and $\mu_u = L$. Given p_2, p_u and μ_u, one can show that $IC_2(\overline{\theta}_L)$ holds, $IC_2(\underline{\theta}_0)$ and $IC_2(\overline{\theta}_H)$ does not hold. Type $\overline{\theta}_H$ keeps. Type $\overline{\theta}_L$ buys new by selling in the resale market. Given p_u and μ_u type $\underline{\theta}_0$ is indifferent between buying from the resale market or not buying at all. Only $\frac{n}{2}$ of type $\underline{\theta}_0$ buys from the resale market, the rest of them do not buy at all. Type $\overline{\theta}_L$ is the only type that buys new: $q_2 = \frac{n}{2}$.

(6) If $\underline{\theta}(\mu_2 - L) < p_2 \leq \underline{\theta}\mu_2$, $p_u = p_2 - \underline{\theta}(\mu_2 - L)$ and $\mu_u = L$. Given k, p_2, p_u and μ_u, one can show that $IC_2(\overline{\theta}_L)$ holds, $IC_2(\underline{\theta}_0)$ holds with equality and $IC_2(\overline{\theta}_H)$ does not hold. Type $\overline{\theta}_H$ keeps. Type $\overline{\theta}_L$ buys new by selling in the resale market. Type $\underline{\theta}_0$ is indifferent between buying new or used. Among type $\underline{\theta}_0$, $\frac{n}{2}$ of them buy from resale market, and the rest buy new. Type $\overline{\theta}_L$ and $1 - \frac{3n}{2}$ of type $\underline{\theta}_0$ buy new: $q_2 = 1 - n$.

(7) If $\overline{\theta}(\mu_2 - H) < p_2 \leq \underline{\theta}(\mu_2 - L)$, $p_u = 0$ and $\mu_u = L$. Given k, p_2, p_u and μ_u, one can show that $IC_2(\overline{\theta}_L)$ and $IC_2(\underline{\theta}_0)$ holds, $IC_2(\overline{\theta}_H)$ does not hold. Type $\overline{\theta}_H$ keeps. Type $\overline{\theta}_L$ buys new by scrapping and type $\underline{\theta}_0$ buys new: $q_2 = 1 - \frac{n}{2}$.

(8) This range is valid only if $k \geq 1+t$. If $p_2 \leq \overline{\theta}(\mu_2 - H)$, $p_u = 0$ and $\mu_u = \mu_1$. Given k, p_2, p_u and μ_u, one can show that $IC_2(\underline{\theta}_0)$, $IC_2(\overline{\theta}_L)$ and $IC_2(\overline{\theta}_H)$ holds. Owners scrap. All types buy new: $q_2 = 1$.

The $\mathbf{p}_1(\mathbf{q}_1, \mathbf{q}_2)$, $\boldsymbol{\pi}_2(\mathbf{q}_1, \mathbf{q}_2)$ and $\boldsymbol{\pi}(\mathbf{q}_1, \mathbf{q}_2)$ Functions

For each range of p_2 given in Table 1, the profit maximizing monopolist chooses the maximum p_2. At this p_2 the incentive constraint of the marginal consumer holds with equality. Given p_2, the $p_1(q_1, q_2)$ are found by using $IC_1(\overline{\theta})$ and $IC_1(\underline{\theta})$ for $q_1 = n$ and $q_1 = 1$, respectively. In Table 2, we give $p_1(q_1, q_2)$, $\pi_2(q_1, q_2) = p_2q_2$ and $\pi(q_1, q_2) = p_1q_1 + \pi_2(q_1, q_2)$ for each p_2.

Table 2 - The $\mathbf{p}_1(\mathbf{q}_1,\mathbf{q}_2)$, $\boldsymbol{\pi}_2(\mathbf{q}_1,\mathbf{q}_2)$ and $\boldsymbol{\pi}(\mathbf{q}_1,\mathbf{q}_2)$ Functions

	$\mathbf{p}_1(\mathbf{q}_1,\mathbf{q}_2)$	$\boldsymbol{\pi}_2(\mathbf{q}_1,\mathbf{q}_2)$	$\boldsymbol{\pi}(\mathbf{q}_1,\mathbf{q}_2)$
If there is no trading in period 1: $\mathbf{q}_1 = \mathbf{0}$			
(1)	–	0	0
(2)	–	$\overline{\theta}nk\mu_1$	$\overline{\theta}nk\mu_1$
(3)	–	$\underline{\theta}k\mu_1$	$\underline{\theta}k\mu_1$
If only type $\overline{\theta}$ consumers buy in period 1: $\mathbf{q}_1 = \mathbf{n}$			
▶ **If** $k < 1 + t\frac{\overline{\theta}}{\overline{\theta}-\underline{\theta}}$			
(4)	$2\overline{\theta}\mu_1$	0	$2\overline{\theta}n\mu_1$
(5)	$\overline{\theta}H + \underline{\theta}L$	$\frac{[\overline{\theta}k-(1-t)(\overline{\theta}-\underline{\theta})]n}{2}\mu_1$	$\frac{[\overline{\theta}k+3t(\overline{\theta}-\underline{\theta})+\overline{\theta}+3\underline{\theta}]n}{2}\mu_1$
(6)	$\frac{2(\overline{\theta}+\underline{\theta})\mu_1-(\overline{\theta}-\underline{\theta})(\mu_2-H)}{2}$	$\underline{\theta}k(1-n)\mu_1$	$\frac{[t(\overline{\theta}-\underline{\theta})-k(\overline{\theta}+\underline{\theta}(1-\frac{2}{n}))+3\overline{\theta}+\underline{\theta})]n}{2}\mu_1$
(7)	$\frac{2\overline{\theta}H-(\overline{\theta}-\underline{\theta})(\mu_2-L)}{2}$	$\frac{\underline{\theta}(k-1+t)(2-n)}{2}\mu_1$	$\frac{[k(2\underline{\theta}-n\overline{\theta})+t(n\overline{\theta}+2\underline{\theta})+3\overline{\theta}n-2\underline{\theta}]}{2}\mu_1$
(8)	$\overline{\theta}\mu_1$	$\overline{\theta}(k-1-t)\mu_1$	$\overline{\theta}(k-1-t+n)\mu_1$
▶ **If** $k \geq 1 + t\frac{\overline{\theta}}{\overline{\theta}-\underline{\theta}}$			
(9)	$2\overline{\theta}\mu_1$	0	$2\overline{\theta}n\mu_1$
(10)	$\overline{\theta}H + \underline{\theta}L$	$\frac{[\overline{\theta}k-(1-t)(\overline{\theta}-\underline{\theta})]n}{2}\mu_1$	$\frac{[\overline{\theta}k+3t(\overline{\theta}-\underline{\theta})+\overline{\theta}+3\underline{\theta}]n}{2}\mu_1$
(11)	$(\overline{\theta}+\underline{\theta})\mu_1$	$[\overline{\theta}(k-1-t)+\underline{\theta}]n\mu_1$	$[\overline{\theta}(k-t)+2\underline{\theta}]n\mu_1$
(12)	$(\overline{\theta}+\underline{\theta})\mu_1$	$\underline{\theta}k(1-n)\mu_1$	$[\underline{\theta}(1-n)k+n(\overline{\theta}+\underline{\theta})]\mu_1$
(13)	$\overline{\theta}\mu_1$	$\underline{\theta}(k-1)\mu_1$	$[\underline{\theta}k-\underline{\theta}+n\overline{\theta}]\mu_1$
If both types buy in period 1: $\mathbf{q}_1 = \mathbf{1}$			
▶ **If** $k < 1 + t\frac{\overline{\theta}+\underline{\theta}(\frac{1-n}{1+n})}{\overline{\theta}-\underline{\theta}}$			
(14)	$2\underline{\theta}\mu_1$	0	$2\underline{\theta}\mu_1$
(15)	$\underline{\theta}H$	$\frac{\overline{\theta}(k-1+t)n}{2}\mu_1$	$\frac{[\overline{\theta}nk+t(2\underline{\theta}+\overline{\theta}n)+2\underline{\theta}-\overline{\theta}n]}{2}\mu_1$
(16)	$\underline{\theta}H$	$\frac{\underline{\theta}(k-1+t)}{2}\mu_1$	$\frac{\underline{\theta}(k+3t+1)}{2}\mu_1$
(17)	$\frac{2\underline{\theta}\mu_1+(\overline{\theta}-\underline{\theta})(\mu_2-H)}{2}$	$\frac{\overline{\theta}(k-1-t)(1+n)}{2}\mu_1$	$\frac{[(k-1-t)(\overline{\theta}(2+n)-\underline{\theta})+2\underline{\theta}]}{2}\mu_1$
(18)	$\underline{\theta}\mu_1$	$\underline{\theta}(k-1-t)\mu_1$	$\underline{\theta}(k-t)\mu_1$
▶ **If** $1 + t\frac{\overline{\theta}+\underline{\theta}(\frac{1-n}{1+n})}{\overline{\theta}-\underline{\theta}} \leq k < 1 + t\frac{\overline{\theta}+\underline{\theta}}{\overline{\theta}-\underline{\theta}}$			
(19)	$2\underline{\theta}\mu_1$	0	$2\underline{\theta}\mu_1$
(20)	$\underline{\theta}H$	$\frac{\overline{\theta}(k-1+t)n}{2}\mu_1$	$\frac{[\overline{\theta}nk+t(2\underline{\theta}+\overline{\theta}n)+2\underline{\theta}-\overline{\theta}n]}{2}\mu_1$
(21)	$\underline{\theta}H$	$\frac{\underline{\theta}(k-1+t)}{2}\mu_1$	$\frac{\underline{\theta}(k+3t+1)}{2}\mu_1$
(22)	$\underline{\theta}(\frac{n\mu_1+H}{1+n})$	$\overline{\theta}(k-1-t)n\mu_1$	$[\overline{\theta}nk+t(\frac{\underline{\theta}}{1+n}-\overline{\theta}n)+\underline{\theta}-\overline{\theta}n]\mu_1$
(23)	$\underline{\theta}(\frac{n\mu_1+H}{1+n})$	$\frac{\underline{\theta}[(k-1)(1+n)+t(1-n)]}{2}\mu_1$	$\frac{\underline{\theta}[k(1+n)+t(1-n+\frac{2}{1+n})+1-n)]}{2}\mu_1$
(24)	$\underline{\theta}\mu_1$	$\underline{\theta}(k-1-t)\mu_1$	$\underline{\theta}(k-t)\mu_1$
▶ **If** $k \geq 1 + t\frac{\overline{\theta}+\underline{\theta}}{\overline{\theta}-\underline{\theta}}$			
(25)	$2\underline{\theta}\mu_1$	0	$2\underline{\theta}\mu_1$
(26)	$\underline{\theta}H$	$\frac{\overline{\theta}(k-1+t)n}{2}\mu_1$	$\frac{[\overline{\theta}nk+t(2\underline{\theta}+\overline{\theta}n)+2\underline{\theta}-\overline{\theta}n]}{2}\mu_1$
(27)	$\underline{\theta}(\frac{n\mu_1+H}{1+n})$	$\overline{\theta}(k-1-t)n\mu_1$	$[\overline{\theta}nk+t(\frac{\underline{\theta}}{1+n}-\overline{\theta}n)+\underline{\theta}-\overline{\theta}n]\mu_1$
(28)	$\underline{\theta}(\frac{n\mu_1+H}{1+n})$	$\frac{\underline{\theta}[(k-1)(1+n)+t(1-n)]}{2}\mu_1$	$\frac{\underline{\theta}[k(1+n)+t(1-n+\frac{2}{1+n})+1-n)]}{2}\mu_1$
(29)	$\underline{\theta}\mu_1$	$\underline{\theta}(k-1-t)\mu_1$	$\underline{\theta}(k-t)\mu_1$

Proof of Proposition 1. For $t = 0$, the demand schedule for the improved product and the respective $p_1(q_1, q_2)$, $\pi_2(q_1, q_2)$ and $\pi(q_1, q_2)$ functions are given in Tables 3 and 4.

Table 3 - The Demand Schedule for the Improved Product with No Quality Uncertainty

$\mathbf{p}_2$	$\mathbf{q}_2$
If there is no trading in period 1: $\mathbf{q}_1 = 0$	
$\bar{\theta}\mu_2 < p_2$	0
$\underline{\theta}\mu_2 < p_2 \leq \bar{\theta}\mu_2$	n
$p_2 \leq \underline{\theta}\mu_2$	1
If only type $\bar{\theta}$ consumers buy in period 1: $\mathbf{q}_1 = \mathbf{n}$	
$\bar{\theta}(\mu_2 - \mu_1) + \underline{\theta}\mu_1 < p_2$	0
$\underline{\theta}\mu_2 < p_2 \leq \bar{\theta}(\mu_2 - \mu_1) + \underline{\theta}\mu_1$	n
$\underline{\theta}(\mu_2 - \mu_1) < p_2 \leq \underline{\theta}\mu_2$	$1 - n$
$p_2 \leq \underline{\theta}(\mu_2 - \mu_1)$	1
If both types buy in period 1: $\mathbf{q}_1 = 1$	
$\bar{\theta}(\mu_2 - \mu_1) < p_2$	0
$\underline{\theta}(\mu_2 - \mu_1) < p_2 \leq \bar{\theta}(\mu_2 - \mu_1)$	n
$p_2 \leq \underline{\theta}(\mu_2 - \mu_1)$	1

Table 4 - The $\mathbf{p}_1(\mathbf{q}_1, \mathbf{q}_2)$, $\boldsymbol{\pi}_2(\mathbf{q}_1, \mathbf{q}_2)$ and $\boldsymbol{\pi}(\mathbf{q}_1, \mathbf{q}_2)$ Functions with No Quality Uncertainty

$\mathbf{p}_1(\mathbf{q}_1, \mathbf{q}_2)$	$\boldsymbol{\pi}_2(\mathbf{q}_1, \mathbf{q}_2)$	$\boldsymbol{\pi}(\mathbf{q}_1, \mathbf{q}_2)$
If there is no trading in period 1: $\mathbf{q}_1 = 0$		
$-$	0	0
$-$	$\bar{\theta}nk\mu_1$	$\bar{\theta}nk\mu_1$
$-$	$\underline{\theta}k\mu_1$	$\underline{\theta}k\mu_1$
If only type $\bar{\theta}$ consumers buy in period 1: $\mathbf{q}_1 = \mathbf{n}$		
$2\bar{\theta}\mu_1$	0	$2\bar{\theta}n\mu_1$
$(\bar{\theta} + \underline{\theta})\mu_1$	$[\bar{\theta}(k-1) + \underline{\theta}]n\mu_1$	$[\bar{\theta}k + 2\underline{\theta}]n\mu_1$
$(\bar{\theta} + \underline{\theta})\mu_1$	$\underline{\theta}k(1-n)\mu_1$	$[\underline{\theta}(1-n)k + n(\bar{\theta} + \underline{\theta})]\mu_1$
$\bar{\theta}\mu_1$	$\underline{\theta}(k-1)\mu_1$	$[\underline{\theta}k - \underline{\theta} + n\bar{\theta}]\mu_1$
If both types buy in period 1: $\mathbf{q}_1 = 1$		
$2\underline{\theta}\mu_1$	0	$2\underline{\theta}\mu_1$
$\underline{\theta}\mu_1$	$\bar{\theta}(k-1)n\mu_1$	$[\bar{\theta}nk + \underline{\theta} - \bar{\theta}n]\mu_1$
$\underline{\theta}\mu_1$	$\underline{\theta}(k-1)\mu_1$	$\underline{\theta}k\mu_1$

In period 2, given q_1 and the demand schedule for the improved product, the monopolist maximizes $\pi_2(q_1, q_2)$. When $q_1 = 0$, π_2 is maximized by $q_2 = n$. When $q_1 = n$, if $k \geq \frac{\bar{\theta} - \underline{\theta}}{\bar{\theta} - \underline{\theta}(\frac{1-n}{n})}$, π_2 is maximized by $q_2 = n$; else it is maximized by $q_2 = 1 - n$. When $q_1 = 1$, π_2 is maximized by $q_2 = n$. In period 1, given the optimal q_2 for each q_1, the monopolist maximizes $\pi(q_1, q_2)$. Among $\pi(0, n)$, $\pi(n, n)$, $\pi(n, 1-n)$ and $\pi(1, n)$, $\pi(n, 1-n)$ is the maximum if $k < \frac{\bar{\theta} - \underline{\theta}}{\bar{\theta} - \underline{\theta}(\frac{1-n}{n})}$; else $\pi(n, n)$ is the maximum. That is in equilibrium if $k < \frac{\bar{\theta} - \underline{\theta}}{\bar{\theta} - \underline{\theta}(\frac{1-n}{n})}$, $(q_1, q_2) = (n, 1-n)$; else $(q_1, q_2) = (n, n)$. Given the equilibrium (q_1, q_2), the prices $p_1(q_1, q_2)$ and $p_2(q_1, q_2)$ can be seen from the Tables 3 and 4. The $p_u = \underline{\theta}\mu_1$ is determined at the maximum willingness to pay of type $\underline{\theta}_0$ to buy a used good. ■

Proof of Proposition 2. With ability to commit, the monopolist maximizes $\pi(q_1, q_2)$. If $k \geq 2(1-\frac{\underline{\theta}}{\overline{\theta}})$, $(q_1, q_2) = (n, n)$ yields the maximum π; else $(q_1, q_2) = (n, 0)$ yields the maximum π. That is in commitment equilibrium: if $k \geq 2(1-\frac{\underline{\theta}}{\overline{\theta}})$, $(q_1, q_2) = (n, n)$; else $(q_1, q_2) = (n, 0)$. ■

Proof of Proposition 3. We show that when $k \geq 4$, in both commitment and no-commitment equilibria: if $t \geq 2\frac{\underline{\theta}}{\overline{\theta}}$, $(q_1, q_2) = (0, n)$; else $(q_1, q_2) = (n, n)$. When $k \geq 4$, the relevant demand schedules for $q_1 = n$ and $q_1 = 1$ are given in Table 1 by (9)-(13) and (15)-(29), respectively. Inspection of $\pi(q_1, q_2)$ functions for the relevant range in Table 2 shows that when $k \geq 4$, $(q_1, q_2) = (0, n)$ yields the maximum π if $t \geq 2\frac{\underline{\theta}}{\overline{\theta}}$ and $(q_1, q_2) = (n, n)$ yields the maximum π if $t < 2\frac{\underline{\theta}}{\overline{\theta}}$. Therefore when $k \geq 4$, in commitment equilibrium: if $t \geq 2\frac{\underline{\theta}}{\overline{\theta}}$, $(q_1, q_2) = (0, n)$; else $(q_1, q_2) = (n, n)$. Inspection of $\pi_2(q_1, q_2)$ functions shows that for all q_1, $\pi_2(q_1, q_2)$ is maximized by selling only to high-valuation consumers, i.e., $q_2 = n$. Comparison of $\pi(0, n)$, $\pi(n, n)$ and $\pi(1, n)$ shows that $\pi(0, n) > \pi(1, n)$, as there is sufficient heterogeneity by assumption (2). Moreover, if $t < 2\frac{\underline{\theta}}{\overline{\theta}}$, $\pi(0, n) < \pi(n, n)$; else $\pi(0, n) \geq \pi(n, n)$. That is when $k \geq 4$, commitment and no-commitment equilibria are identical. ■

Proof of Proposition 4. We show that when $k < 2(1-\frac{\underline{\theta}}{\overline{\theta}})$ commitment and no-commitment equilibria are identical if $t \geq 1-\frac{k\min\left\{\frac{\overline{\theta}}{3}, \overline{\theta}-2\underline{\theta}(\frac{1-n}{n})\right\}}{\overline{\theta}-\underline{\theta}}$; else they differ. Suppose $k < 2(1-\frac{\underline{\theta}}{\overline{\theta}})$. Inspection of $\pi(q_1, q_2)$ given in Table 2 shows that if $t \geq 1-\frac{k\overline{\theta}}{3(\overline{\theta}-\underline{\theta})}$, $(q_1, q_2) = (n, \frac{n}{2})$ yields the maximum π; else commitment to no trading, i.e., $(q_1, q_2) = (n, 0)$ yields the maximum π. That is when $k < 2(1-\frac{\underline{\theta}}{\overline{\theta}})$, in commitment equilibrium: if $t \geq 1-\frac{k\overline{\theta}}{3(\overline{\theta}-\underline{\theta})}$, $(q_1, q_2) = (n, \frac{n}{2})$; else $(q_1, q_2) = (n, 0)$. It is clear that when the commitment solution requires no trading in period 2, there is a time inconsistency problem. That is, commitment and no-commitment equilibria differ when $t < 1-\frac{k\overline{\theta}}{3(\overline{\theta}-\underline{\theta})}$.

Now we focus on the range $t \geq 1-\frac{k\overline{\theta}}{3(\overline{\theta}-\underline{\theta})}$ at which $(q_1, q_2) = (n, \frac{n}{2})$ yields the maximum π. In this range, given that $q_1 = n$, if π_2 is maximized by $q_2 = \frac{n}{2}$, commitment and no-commitment equilibria are identical; otherwise they differ. Suppose $q_1 = n$ and $t \geq 1-\frac{k\overline{\theta}}{3(\overline{\theta}-\underline{\theta})}$. If there is sufficient heterogeneity in valuations such that $\overline{\theta}n \geq 3(1-n)\underline{\theta}$ (i.e., $1-\frac{k\overline{\theta}}{3(\overline{\theta}-\underline{\theta})} \geq 1-\frac{k[\overline{\theta}-2\underline{\theta}(\frac{1-n}{n})]}{\overline{\theta}-\underline{\theta}}$), π_2 is maximized by $q_2 = \frac{n}{2}$. Else if heterogeneity is lower, $\pi_2(n, \frac{n}{2}) < \pi_2(n, 1-n)$ when $1-\frac{k\overline{\theta}}{3(\overline{\theta}-\underline{\theta})} \leq t < 1-\frac{k[\overline{\theta}-2\underline{\theta}(\frac{1-n}{n})]}{\overline{\theta}-\underline{\theta}}$. That is commitment and no-commitment equilibria are identical if $t \geq 1 - \frac{k \min\left\{\frac{\overline{\theta}}{3}, \overline{\theta} - 2\underline{\theta}\left(\frac{1-n}{n}\right)\right\}}{\overline{\theta} - \underline{\theta}}$; else they differ. ■

Proof of Proposition 5. We show that when $2(2 - \frac{\underline{\theta}}{\overline{\theta}}) \leq k < 4$ commitment and no-commitment equilibria are identical if $t \leq \frac{\overline{\theta}(k-1) - 3\underline{\theta}}{3(\overline{\theta} - \underline{\theta})}$; else they differ. Comparison of the $\pi(q_1, q_2)$ functions given in Table 2 shows that when $2(2 - \frac{\underline{\theta}}{\overline{\theta}}) \leq k < 4$, the π is maximized by $(q_1, q_2) = (n, n)$ if $t < 2\frac{\underline{\theta}}{\overline{\theta}}$, by $(q_1, q_2) = (0, n)$ if $2\frac{\underline{\theta}}{\overline{\theta}} \leq t < \frac{\overline{\theta}(k-1) - 3\underline{\theta}}{3(\overline{\theta} - \underline{\theta})}$ and by $(q_1, q_2) = (n, \frac{n}{2})$ if $\frac{\overline{\theta}(k-1) - 3\underline{\theta}}{3(\overline{\theta} - \underline{\theta})} \leq t$. If $q_1 = 0$ or $q_1 = n$, $\pi_2(q_1, q_2)$ is maximized by $q_2 = n$. That is commitment and no-commitment equilibria are identical if and only if $t \leq \frac{\overline{\theta}(k-1) - 3\underline{\theta}}{3(\overline{\theta} - \underline{\theta})}$. ■

Proof of Propositions 6 and 7. Suppose $t \geq 2\frac{\underline{\theta}}{\overline{\theta}}$ and $k \geq \max\left\{\frac{(1+3t)\overline{\theta} - (1+t)\underline{\theta}}{\overline{\theta}}, 2\right\}$. A comparison of $\pi_2(q_1, q_2)$ functions given in Table 2 shows that when $q_1 = 0$ or $q_1 = n$, π_2 is maximized by selling only to high-valuation consumers, i.e., $q_2 = n$. When $q_1 = 1$, if $k \geq 1 + 3t$, π_2 is maximized by $q_2 = n$; else it is maximized by $q_2 = \frac{n}{2}$. In period 1, given the optimal q_2 for each q_1, the monopolist maximizes $\pi(q_1, q_2)$. Among $\pi(0, n)$, $\pi(n, n)$, $\pi(1, n)$ and $\pi(1, \frac{n}{2})$, $\pi(0, n)$ is the maximum. That is with no-commitment, there is no trading in period 1. With ability to commit, the monopolist maximizes $\pi(q_1, q_2)$. If $k \geq \max\left\{\frac{(1+3t)\overline{\theta} + 3(1-t)\underline{\theta}}{\overline{\theta}}, 2\right\}$, $(q_1, q_2) = (0, n)$ yields the maximum π and if $\max\left\{\frac{(1+3t)\overline{\theta} - (1+t)\underline{\theta}}{\overline{\theta}}, 2\right\} \leq k < \max\left\{\frac{(1+3t)\overline{\theta} + 3(1-t)\underline{\theta}}{\overline{\theta}}\right\}$, $(q_1, q_2) = (n, \frac{n}{2})$ yields the maximum π. That is commitment and no-commitment equilibria are identical if $k \geq \max\left\{\frac{(1+3t)\overline{\theta} + 3(1-t)\underline{\theta}}{\overline{\theta}}, 2\right\}$, and if $\max\left\{\frac{(1+3t)\overline{\theta} - (1+t)\underline{\theta}}{\overline{\theta}}, 2\right\} \leq k < \max\left\{\frac{(1+3t)\overline{\theta} + 3(1-t)\underline{\theta}}{\overline{\theta}}\right\}$, commitment and no-commitment equilibria differ. ■

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