Dependence Structures in Chinese and U.S. Financial Markets: A Time-varying Conditional Copula Approach

Jian Hu[1]

Southern Methodist University

May 11, 2008

Abstract

In this paper, we use Time-Varying Conditional Copula approach (TVCC) to model Chinese and U.S. stock markets' dependence structures with other major stock markets around the world. $AR-GARCH-t$ model is used to examine the margins, while two copula models are employed to analyze the joint distributions. In this pairwise analysis, both constant and time-varying conditional dependence parameters are estimated by two-step maximum likelihood method. Dependence depends on past realizations and some of dependence parameters are persistent while others are quite volatile. A comparative analysis of dependence structures in Chinese and U.S. stock markets is also provided. There are three main findings: Firstly, the time-varying dependence model, though gives more information on the dependence change over time, doesn't always perform better than constant dependence model. This result hasn't been reported in the literature. Secondly, notwithstanding previous research extensively reports that the dependence between stock markets tend to be higher during market downturn than during market upturn, we find a counter-example that dependence is much higher during market upturn than during market downturn. Finally, dependence structures of Chinese and U.S. stock markets are sustantially different from each other.

JEL classification: C51; F36; G15; P52
Key words: AR-GARCH-t model; Time-varying conditional copula; Dependence structure; Stock market

1 Introduction

The dependence of financial markets has been a heatedly-debated issue to financial economists in both academia and investment industry as it has consequences for the identification of opportunities for and significant barriers to

[1]Department of Economics, Southern Methodist University, 3300 Dyer Street, Dallas TX 75275. Tel: +1 214 718 2908. Email: jianh@smu.edu. I would like to thank Mehtabul Azam, Nathan Balke, Jim Cooley, Tom Fomby, Jong Kim, Essie Maasoumi, Daniel Millimet and other seminar participants at SMU for their valuable comments and suggestions.

international portfolio management with important implications for asset allocation and pricing; see Bartram and Dufey (2001) among others. The widely used linear dependence measure is too simple to correctly characterize financial return distributions under certain conditions. As Jondeau and Rockinger (2006) point out, when financial returns are non-normal, it is impossible to specify the multivariate distribution relating two or more return series. The copula may be one possible way to overcome drawbacks of linear dependence measure.

Previous research has investigated how the correlation between stock market returns varies over time. There exists significant asymmetric dependence. For example, Longin & Solnik (1995) examine correlations between stock markets over a long time period of using the constant conditional correlation (CCC) model proposed by Bollerslev (1990). They find that correlations are generally higher during more volatile periods and depend on several economic variables. After that, Longin and Solnik (2001) find that international stock markets are more correlated in bear markets, using extreme correlation with a copula model. Ang and Chen (2002) propose a test for asymmetric correlation by comparing empirical and model-based conditional correlations. Patton (2004) finds dependence asymmetry of financial returns both in the marginal distributions and in the dependence structure. Patton (2006a) and Patton (2006b) develop theory of conditional copulas and employ time-varying copula models to analyze two foreign exchange series. Jondeau & Rockinger (2006) model financial returns with time-varying Skewed-t GARCH models and then use a time-varying or a switching Gaussian or Student's t copula for the dependence between countries. Okimoto (2007) estimates regime-switching copulas for pairs of US-UK and other G7 countries. Rodriguez (2007) adopts the copula model with Markov switching parameters and finds evidence of changing dependence structures during periods of financial turmoil, and increased tail dependence and asymmetry in times of high volatility characterize Asian countries within a relatively short time period.

As the largest emerging market in the world, China has been experiencing rapid economic growth in last two decades, which leads to fast growing Chinese stock market. Unfortunately, Chinese financial market attracts less attention in academics. In late 1997, the Asian countries experienced a huge financial crisis, however, China survived this turbulence. This financial crisis attracted more attention to dependence between financial markets. Kim (2005) finds that there exist some differences in the time path of dependence among Asian countries. The question is whether the degree of dependence between China and other countries is lower than that between other countries, so that China can uniquely kept away from this crisis. What is dependence structure between Chinese stock market and other major stock markets around the world? It is also interesting to compare dependence structures with the rest of the world in Chinese and U.S. stock markets.

This study is devoted to Chinese and U.S. stock markets. The purposes of this study are the following. First, we investigate the different dependence structures between Chinese stock market and each major stock market as well as those between U.S. stock market and others. It is, to my best knowledge, the first attempt to examine the dependence structure between Chinese financial market and other major markets. Another new finding is that it is possible to have higher dependence during market upturn than during market downturn, which has not been documented in the financial contagion literature. Secondly, we try to examine the dynamics of general dependence and tail dependence using time-varying conditional copula. We contribute to the literature in the sense that we find that the time-varying model doesn't always perform better than their constant peers. Finally, a comparative analysis between China-related models and U.S.-related models is implemented and some suggestions for practitioners are given. We find that there may be a general level of dependence among financial markets in developed countries and the dependence among western financial markets have a more groupwise flavor.

This paper is organized as follows. Next section provides a brief review of copulas and conditional copulas. In section 3, we discuss the model specification, including the choice of estimation strategy and specific marginal and copula models. Section 4 presents estimation results for both marginal and copula models. Section 5 concludes.

2 Theory of Conditional Copula

2.1 Copula

It is necessary to understand what is copula (unconditional) before we discuss conditional copula. For simplicity, we will focus on only bivariate copulas even though multivariate case can be immediately extended. Suppose we have two random variables Y_1 and Y_2. Then the joint distribution function can be written as:

$$F(y_1, y_2) = \Pr(Y_1 \leq y_1, Y_2 \leq y_2) \tag{1}$$

where y_1 and y_2 denote the realizations of random variables Y_1 and Y_2, respectively.

A copula is actually a multivariate joint distribution. It allow us to decompose a joint distribution into its marginal distribution and its dependence function, i.e. copula. We may construct the copula function by transforming the random variables Y_1 and Y_2 to uniform marginal distribution (CDF), i.e. F_1, F_2. Mathematically,

$$\begin{aligned} F(y_1, y_2) &= \Pr(F_1(Y_1) \leq F_1(y_1), F_2(Y_2) \leq F_2(y_2)) \\ &= C(F_1(y_1), F_2(y_2)) \end{aligned} \tag{2}$$

A complete and formal definition of copulas can be found in Nelsen (2006). Also, Joe(1997) provided many nice properties of various copula families.

2.2 Conditional Copula

Patton (2006a) summarizes the conditional copula theory. We give a brief review here. Similar to unconditional case, we have two random variables Y_1 and Y_2. We introduce conditioning variable/vector W. Let $F_{Y_1Y_2|W}$ denote the conditional distribution of (Y_1, Y_2) given W, and let the conditional marginal distributions of $Y_1|W$ and $Y_2|W$ be denoted $F_{Y_1|W}$ and $F_{Y_2|W}$, respectively. We assume that $F_{Y_1|W}$, $F_{Y_2|W}$ and $F_{Y_1Y_2|W}$ are all continuous for simplicity.[2] The Theorem 1 on conditional copula in Patton (2006a) is reproduced below:

Theorem 1 *Let $F_{Y_1|W}(\cdot|w)$, $F_{Y_2|W}(\cdot|w)$ be the conditional distribution of $Y_1|W = w$ and $Y_2|W = w$, respectively, $F_{Y_1Y_2|W}(\cdot|\omega)$ be the joint conditional distribution of $(Y_1, Y_2)|W = w$ and ω be the support of W. Assume that $F_{Y_1|W}(\cdot|w)$ and $F_{Y_2|W}(\cdot|w)$ are continuous in y_1 and y_2 for all $w \in \omega$. Then there exists a unique conditional copula $C(\cdot|\omega)$ such that*

$$\begin{aligned} F_{Y_1Y_2|W}(y_1,y_2|\omega) &= C(F_{Y_1|W|}(y_1|w), F_{Y_2|W}(y_2|w)|w) \\ &= C(u,v) \qquad (3) \\ \forall(y_1,y_2) &\in \bar{R} \times \bar{R} \text{ and } w \in \omega \qquad (4) \end{aligned}$$

where $u = F_{Y_1|W}(y_1|w)$ and $v = F_{Y_2|W}(y_2|w)$ are realizations of $U \equiv F_{Y_1|W}(Y_1|w)$ and $V \equiv F_{Y_2|W}(Y_2|w)$ given $W = w$.

Theorem 1 is nothing but an extension of Sklar's Theorem (1959). U and V are the conditional "probability integral transforms" of Y_1 and Y_2. Fisher (1932) and Rosenblatt (1952) prove that U and V follow the $Unif(0,1)$ distribution, regardless of the original distributions. This is where the nice properties of copulas come from. Patton (2002) shows that a conditional copula has the properties of an unconditional copula. There are many copula families. In next section, we will talk about specific copulas used in our model.

3 Model Specification

3.1 Estimation Strategy

It has been widely accepted that the financial time series follow Student's t distribution. Also, in our case, the serial correlation and heteroskedasticity are tested, hence the standard $AR(p) - GARCH(1,1) - t$ model is used to model each marginal distribution. After estimating the marginal distribution, we will estimate copula parameter using maximum likelihood method. Let $u \equiv F_{Y_1|W}(y_1|w;\theta_1)$ and $v \equiv F_{Y_2|W}(y_2|w;\theta_2)$, where θ_1 and θ_2 are the vectors of parameters of each margins (or the coefficients of conditioning variable/vector W). Given $C(u,v;\delta) = C(F_{Y_1|W}(y_1|w;\theta_1), F_{Y_2|W}(y_2|w;\theta_2);\delta)$, the copula density is

[2] This assumption is not necessary for the properties of copulas to hold. See Nelsen (2006).

$$c(u,v;\delta)=\frac{\partial^2 C(u,v;\delta)}{\partial u \partial v} \tag{5}$$

Hence the joint density of an observation $(y_{1,t},y_{2,t})$ is

$$\begin{aligned} c(y_{1,t},y_{2,t};\delta) &= \frac{\partial^2 C(u_t,v_t;\delta)}{\partial u_t \partial v_t}\cdot\frac{\partial u_t}{\partial y_{1,t}}\cdot\frac{\partial v_t}{\partial y_{2,t}} \\ &= c(u_t,v_t;\delta)\cdot f_{Y_1|W}(y_{1,t}|w;\theta_1)\cdot f_{Y_2|W}(y_{2,t}|w;\theta_2) \qquad (6) \end{aligned}$$

Therefore, the log-likelihood of a sample can be written as

$$\begin{aligned} L(y_{1,t},y_{2,t};\delta,\theta_1,\theta_2) &= \sum_{t=1}^{T}\ln[c(u_t,v_t;\delta)\cdot f_{Y_1|W}(y_{1,t}|w;\theta_1)\cdot f_{Y_2|W}(y_{2,t}|w;\theta_2)] \qquad (7) \\ &= \sum_{t=1}^{T}\ln[c(F_{Y_1|W}(y_{1,t}|w;\theta_1),F_{Y_2|W}(y_{2,t}|w;\theta_2);\delta) \qquad (8) \\ &\quad \cdot f_{Y_1|W}(y_{1,t}|w;\theta_1)\cdot f_{Y_2|W}(y_{2,t}|w;\theta_2)] \qquad (9) \\ &= \sum_{t=1}^{T}\ln c(F_{Y_1|W}(y_{1,t}|w;\theta_1),F_{Y_2|W}(y_{2,t}|w;\theta_2);\delta) \qquad (10) \\ &\quad +\sum_{t=1}^{T}\ln f_{Y_1|W}(y_{1,t}|w;\theta_1)+\sum_{t=1}^{T}\ln f_{Y_2|W}(y_{2,t}|w;\theta_2) \qquad (11) \\ &= L_C+L_{Y_1}+L_{Y_2} \qquad (12) \end{aligned}$$

where $L_C(y_{1,t},y_{2,t};\delta,\theta_1,\theta_2)=\sum_{t=1}^{T}\ln c(F_{Y_1|W}(y_{1,t}|w;\theta_1),F_{Y_2|W}(y_{2,t}|w;\theta_2);\delta)$, $L_{Y_1}(y_{1,t};\theta_1)=\sum_{t=1}^{T}\ln f_{Y_1|W}(y_{1,t}|w;\theta_1)$, and $L_{Y_2}(y_{2,t};\theta_2)=\sum_{t=1}^{T}\ln f_{Y_2|W}(y_{2,t}|w;\theta_2)$ are the individual log-likelihood functions of copula and two margins.

There are two parametric estimation methods available for copula modeling. One is one-step procedure, the other one is two-step procedure. The one-step procedure is to estimate all parameters of margins and copula at one time. Then maximum likelihood estimation yields $\hat{\theta}=(\hat{\delta},\hat{\theta}_1,\hat{\theta}_2)$, such that

$$\hat{\theta}=\arg\max L(y_{1,t},y_{2,t};\delta,\theta_1,\theta_2) \tag{13}$$

However, in some situations, the maximum likelihood estimation may be difficult to conduct due to too many parameters or just the complexity of the model. As Jondeau and Rockinger (2006) point out, the time-varying dependence parameter may be a convoluted expression of many parameters, hence an analytical expression of the gradient of the likelihood might not exist. Therefore, only numerical gradients may be computable, implying a dramatic slowing down of the numerical procedure. In such a case, a two-step maximum likelihood estimation procedure, also known as Inference Functions for Margins method (IFM) is necessary. In this paper, we use $AR(p)-GARCH(1,1)-t$ model to estimate margins, which lead to many parameters. We also allow the dependence parameters to vary over time, hence the number of parameters increases again. Due to the large number of parameters and the complexity of our model, we choose two-step estimation strategy. This approach, proposed by Shih and Louis (1995) and Joe and Xu (1996), is nothing but the maximum likelihood estimation of the dependence parameter given the estimated marginal distributions. In first step, the parameters in the marginal distributions are estimated

$$\tilde{\theta}_k = \arg\max L_{Y_k}(y_{k,t};\theta_k) \text{ for k=1,2} \tag{14}$$

In second step, copula parameter is estimated given $\tilde{\theta}_1$ and $\tilde{\theta}_2$ from first step

$$\tilde{\delta} = \arg\max L_C(y_{1,t}, y_{2,t}; \delta, \tilde{\theta}_1, \tilde{\theta}_2) \tag{15}$$

Note that the density estimation of each margin doesn't affect the estimation of copula parameter in the second step because each margin is actually estimated in the first step and hence constant in the second step. Therefore, we only need to maximize $L_C(y_{1,t}, y_{2,t}; \delta, \tilde{\theta}_1, \tilde{\theta}_2)$ to get the estimate of copula parameter. Patton (2006b) has proved that this two-step estimation produces normal and asymptotically efficient parameter estimates.

3.2 Marginal Model

To estimate bivariate distribution, we need to make assumption about each univariate marginal distribution first. In this study, we assume each marginal distribution follows $AR(p)-GARCH(1,1)-t$ process. This is standard model for financial returns introduced by Bollerslev (1987), which is widely used in literature; see Patton (2002, 2006a) Jondeau and Rockinger (2006) and Hu (2006) among others. Mathematically,

$$y_{i,t} = \alpha_i + \sum_{j=1}^{p} \beta_j y_{i,t-j} + \varepsilon_{i,t} \text{ for i=1,2} \tag{16}$$

$$\sqrt{\frac{\nu}{\sigma_{i,t}^2(\nu-2)}} \cdot \varepsilon_{i,t}|I_{t-1} \sim t(\nu) \tag{17}$$

$$\sigma_{i,t}^2 = a_i + b_i\sigma_{i,t-1}^2 + c_i\varepsilon_{i,t-1}^2 \tag{18}$$

where $y_{i,t}$ represents univariate stock index return series, α_i is the conditional mean for ith series, $\varepsilon_{i,t}$ is error term in conditional mean equation, $\sigma_{i,t}^2$ is variance, ν is the degree of freedom of Student's t distribution, I_{t-1} is the information set at time $t-1$. We can consider this information set as conditioning variable/vector W. The standardized residuals follow Student's t distribution with degree of freedom ν.

3.3 Copula Model

We will mainly focus on Normal copula and Symmetrized Joe-Clayton copula since the former one is a good model to measure general dependence and the latter one is good at modeling both upper and lower tail dependence. These two types of copula models will give us a full picture of dependence structures for financial returns. However, for comparison purpose, we will estimate the models using different copula functions. These results will be discussed in next section.

3.3.1 Normal (Gaussian) Copula

The first copula of interest is Normal copula, which is the dependence function associated with bivariate normality, and can be written as:

$$C^N(u,v;\rho) = \int_{-\infty}^{\Phi^{-1}(u)} \int_{-\infty}^{\Phi^{-1}(v)} \frac{1}{2\pi\sqrt{(1-\rho^2)}} \exp\left\{\frac{-(r^2 - 2\rho rs + s^2)}{2(1-\rho^2)}\right\} dr\ ds \tag{19}$$

where Φ^{-1} is the inverse of the standard normal CDF, ρ is the correlation coefficient.

In this paper, we assume that the functional form of copula is fixed throughout the sample period while the dependence parameter is time-varying following some evolution equation. We follow Patton (2006a) 's work to assume the following evolution dynamics for ρ_t:

$$\rho_t = \Lambda\left(\omega_\rho + \beta_\rho \cdot \rho_{t-1} + \alpha_\rho \cdot \frac{1}{10}\sum_{j=1}^{10}[\Phi^{-1}(u_{t-j}) \cdot \Phi^{-1}(v_{t-j})]\right) \tag{20}$$

where $\Lambda(x) = \dfrac{(1-e^{-x})}{(1+e^{-x})}$ is the modified logistic transformation, aiming to keep ρ_t within $(-1,1)$ interval. Here we assume that the copula dependence parameter follows an $ARMA(1,10)$-type process, in which the autoregressive term $(\beta_\rho \cdot \rho_{t-1})$ captures persistence effect and the last term $(\alpha_\rho \cdot \frac{1}{10}\sum_{j=1}^{10}[\Phi^{-1}(u_{t-j}) \cdot \Phi^{-1}(v_{t-j})])$ captures variation effect in dependence. The functional form of this evolution equation can be changed since is hard to know what does the dynamics of dependence look like. Here we follow Patton (2006a) to make our results comparable to previous research.[3]

3.3.2 Symmetrized Joe-Clayton Copula

The second copula used in our study is Symmetrized Joe-Clayton (SJC) copula proposed by Patton (2006a), which is basically a slight modification of original

[3] Actually, we have tried several different evolution equations here, such as including lag 2 autoregressive term or replacing 10 with 20 in last term, but there is no significant improvement in our maximum likelihood estimation.

Joe-Clayton copula. Joe-Clayton copula (also known as "BB7" copula) proposed by Joe (1997) is a Laplace transformation of Clayton's copula . It is defined as

$$C^{JC}(u,v;\tau^U,\tau^L) = 1-(1-\left\{[1-(1-u)^{\kappa}]^{-\gamma}+[1-(1-v)^{\kappa}]^{-\gamma}-1\right\}^{-1/\gamma})^{1/\kappa} \tag{21}$$

where $\kappa = 1/\log_2(2-\tau^U)$, $\gamma = -1/\log_2(\tau^L)$ and $\tau^U \in (0,1]$, $\tau^L \in (0,1]$.

Unlike normal copula, there are two tail dependence parameters, τ^U and τ^L, in this copula function. Upper tail dependence is defined as

$$\tau^U = \lim_{\varepsilon\to 1}\Pr[U>\varepsilon|V>\varepsilon] = \lim_{\varepsilon\to 1}\Pr[V>\varepsilon|U>\varepsilon] = \lim_{\varepsilon\to 1}(1-2\varepsilon+C(\varepsilon,\varepsilon)/(1-\varepsilon) \tag{22}$$

If this limit exists, the copula shows upper tail dependence when $\tau^U \in (0,1]$ and no tail dependence when $\tau^U = 0$. Similarly, we can define lower tail dependence as

$$\tau^L = \lim_{\varepsilon\to 0}\Pr[U\leqslant\varepsilon|V\leqslant\varepsilon] = \lim_{\varepsilon\to 0}\Pr[V\leqslant\varepsilon|U\leqslant\varepsilon] = \lim_{\varepsilon\to 0}\Pr C(\varepsilon,\varepsilon)/\varepsilon \tag{23}$$

If this limit exists, the copula shows lower tail dependence when $\tau^L \in (0,1]$ and no tail dependence when $\tau^L = 0$.

By construction, the Joe-Clayton copula always gives asymmetric tail dependence even if two tail dependence measures are in fact equal. In order to overcome this shortcoming, we will use Symmetrized Joe-Clayton copula, which is given by

$$C^{SJC}(u,v;\tau^U,\tau^L) = 0.5\cdot(C^{JC}(u,v;\tau^U,\tau^L)+C^{JC}(1-u,1-v;\tau^U,\tau^L)+u+v-1) \tag{24}$$

where C^{JC} is Joe-Clayton copula.

The advantage of SJC copula is it can be symmetric when $\tau^U = \tau^L$, whereas the original Joe-Clayton copula still contains asymmetry even though tail dependence is symmetric (when $\tau^U = \tau^L$). So the SJC copula is virtually a generalized version of Joe-Clayton copula allowing tail dependence to be either asymmetric or symmetric. This property makes SJC copula more attractive for empirical work because of its generality. Gumbel and Clayton copula also capture tail dependence, however, empirical research shows that estimating Gumbel and Clayton copulas separately does not produce much different results from estimating Joe-Clayton copula alone, as reported by Kim (2005).

Tail dependence refers to the level of dependence in the upper-right-quadrant tail and lower-left-quadrant tail of a multivariate distribution, hence it is an appropriate measure of dependence of extreme events. This nice property makes it very useful to examine the joint extreme events in financial returns during high volatility or market crashing periods. One explanation of tail dependence in our paper is a probability measure of joint extreme values in two financial markets given one extreme value in one of the two markets.

Similar to the dynamics of ρ_t in Normal copula, we propose the following evolution equations for τ^U and τ^L, respectively (see Patton (2006) for more detailed explanation)

$$\tau_t^U = \Pi\left(\omega_U + \beta_U \cdot \tau_{t-1}^U + \alpha_U \cdot \frac{1}{10}\sum_{j=1}^{10}|u_{t-j} - v_{t-j}|\right) \tag{25}$$

$$\tau_t^L = \Pi\left(\omega_L + \beta_L \cdot \tau_{t-1}^L + \alpha_L \cdot \frac{1}{10}\sum_{j=1}^{10}|u_{t-j} - v_{t-j}|\right) \tag{26}$$

where Π is the logistic transformation, used to keep τ^U and τ^L within $(0,1)$ interval. This dynamics is again $ARMA(1,10)$-type model with an autoregressive term $(\beta \cdot \tau_{t-1})$ and a forcing variable $(\alpha \cdot \frac{1}{10}\sum_{j=1}^{10}|u_{t-j} - v_{t-j}|)$, where the autoregressive term represents persistence effect and the forcing variable captures variation in dependence. Note that we assume that τ^U and τ^L evolve in the same pattern even thought it is possible that they follow different dynamics. We use 10 lags in the forcing variable to make the evolution equation comparable with that of Normal copula.

4 Empirical Results

4.1 Data Description

We examine the interaction between Chinese/U.S. stock indices and each of other stock indices. The labels are "CHN" for Shanghai Stock Exchange Composite from China, "DEU" for DAX from Germany, "FRA" for CAC 40 from France, "GBR" for FTSE 100 from the United Kingdom, "HKG" for Hang Seng Stock Exchange Index from Hong Kong, "JPN" for Nikkei 225 from Japan and "USA" for S&P 500 from the United States. I use daily stock indices from Datastream from January 2nd, 1991 to December 31st, 2007. The sample runs 17 years and covers 4434 data points. Table 1 gives summary statistics on stock market returns. As usual, returns are defined as 100 times log-difference of index values, where P_t is the value of the index at time t. This reduces the sample by one record, yielding 4433 observations.

$$R_t = 100 \times \log P_t / P_{t-1} \tag{27}$$

We have the following findings: Firstly, in Panel A of Table 1, the average return of Chinese stock market is the highest one followed by Hong Kong market. In particular, Japanese stock market shows bad performance considering the negative average return. According to standard deviation, the most volatile stock market is Chinese market and the next is Hong Kong, while the less volatile market is USA. Means of each series are very small relative to their standard deviations. Most of markets exhibit slight negative skewness (i.e. left-skewed) except China and Japan. China even reaches 6.05, which implies that the distribution is highly right-skewed. All these results show that the empirical distribution of returns exhibit fatter tails than normal distribution. We also find significant kurtosis in each return series. China displays extremely high kurtosis. This high kurtosis means more of the variance is due to infrequent extreme deviations.

Secondly, in Panel B of Table 1, all series strongly rejects the Jarque-Bera test, showing non-normality of unconditional distribution of each series. This is one of the reasons why multivariate normal distribution would be inappropriate. We perform LM test to examine whether the squared return is serially correlated up to lag 1, 5 and 10. This statistic clearly indicates that ARCH effects are likely to be found in all market returns.[4] Even if there is one insignificant statistic of ARCH LM(1) test for Chinese stock market, it is significant at 5% level using lag 5 and 10. Ljung-Box autocorrelation test with correction for heteroskedesticity is also implemented at lag 1, 5 and 10, implying most of return series are serially correlated, at least at one of the lag orders.[5]

Finally, in Panel C of Table 1, the unconditional correlation matrix indicates that a rather high dependence between geographically close countries is expected. The correlations between DEU, FRA and GBR are relatively higher than those of other pairs. There is some extra findings on the relationship between distance and stock market correlation in this paper. We will discuss this issue in the further research section. Unconditional correlations between China and other countries are small, but whether conditional correlations are small or not is still unknown. The linear unconditional correlations in China-related pairs range from -0.0158 to 0.0511. The ranking from the highest to the lowest is CHN/HKG, CHN/JPN, CHN/DEU, CHN/FRA, CHN/GBR, CHN/USA. The ranking of Spearman correlations remains the same as that of linear correlations. Most of Spearman correlations are less than linear correlations except CHN/HKG, which actually increases by 53% (from 0.051 to 0.078). For the U.S.-related pairs, the linear correlations range from -0.016 to 0.455. The ranking of linear correlations in descending order is USA/DEU, USA/FRA, USA/GBR, USA/HKG, USA/JPN, USA/CHN. However, the ranking of Spearman correlation changes into USA/FRA, USA/GBR, USA/DEU, USA/JPN, USA/HKG, USA/CHN in descending order. Most of Spearman correlations are less than their linear correlations except USA/JPN, which actually increases 7.3% (from 0.109 to 0.117). The linear correlation is only one way to measure dependence. In order to use it correctly, two conditions must be satisfied: (1) the data in the pairs both come from normal distributions and (2) the data are at least in the category of equal interval data. The first condition is evidently violated in our case, so linear correlation is not effective way to evaluate dependence. Another possibility is to use Spearman (Rank) correlation coefficient. Copula

[4] Other lag orders are also used to perform this test, almost all of them show significant ARCH effect. The results are available upon request.

[5] Other lag orders are also used to perform this test, most of them show statistically significant serial correlation at 5% significance level. The results are available upon request.

dependence parameter is easily transformed to this rank correlation. According to Table 1, Spearman correlations are a little less than the linear correlation for most pairs.

[Table 1]

4.2 Estimation of the Marginal Models

We use two-step estimation methods in this paper due to a great number of parameters in time-varying models. First, we select the different lag orders model for the mean equations based on Akaike Information Criterion (AIC), keeping the conditional variance equation as GARCH(1,1) for each country. We choose AR(17) for CHN, AR(6) for DEU, AR(7) for FRA, AR(6) for GBR, AR(3) for HKG, AR(1) for JPN, and AR(7) for USA. The results for the marginal distributions are reported in Table 2.

[Table 2]

We then conduct model misspecification test by suggestion of Diebold, Gunther and Tay (1998). They suggested a less formal but very useful test. They examined the correlograms of $(z-\bar{z}), (z-\bar{z})^2, (z-\bar{z})^3$, and $(z-\bar{z})^4$, where z is the probability integral transforms (u and v in our study). Each moment reveals dependence operative through the conditional mean, conditional variance, conditional skewness, and conditional kurtosis. Figure 1 presents the test results. In the $AR-GARCH-t$ case, the correlograms show that there is no serial correlation in the first four moments with few exceptions. So we can conclude that our marginal distribution models for all countries are correctly specified. Put differently, our marginal models are adequate for financial returns.

[Figure 1]

4.3 Estimation of the Copula Models

We present the estimation results of the dependence structure using a variety of copula models. Table 3 and 4 summarize the log-likelihood value of each copula estimation with a constant parameter. In Table 3, we can see that Gumbel and Rotated Gumbel copula produce higher log-likelihood except CHN/HKG pair and SJC copula is not bad. In Table 4, Student's t copula produce higher log-likelihood in USA/DEU, USA/FRA, and USA/GBR pairs while SJC copula produce highest log-likelihood in USA/HKG and USA/JPN pairs. Since these copula models are not nested in each other, it is hard to say which one provides the best fit in terms of log-likelihood. Given that our purpose is to examine general dependence as well as tail dependence (including both upper and lower tail dependence), we will stick to Normal and SJC copulas. The Normal copula parameter is the alternative measure of linear correlation, while the SJC copula parameters capture both upper and lower tail dependence.

[Table 3 and 4]

4.3.1 Results for China-related Copula Models

Table 5 reports China-related Normal and Symmetrized Joe-Clayton (SJC) copula parameter estimates of both constant and time-varying cases for the purpose of comparison.

Normal Copula In Panel A1, the constant dependence measures are significantly different from linear correlations reported in Table 1 Panel C. Specifically, the CHN/DEU pair decreases by 29% from 0.007 to 0.005, the CHN/FRA pair decreases from 0.002 to -0.004 with the change of sign, the CHN/GBR pair is less negative (from -0.002 to -0.001), the CHN/HKG pair increases by 69% from 0.051 to 0.086, the CHN/JPN pair increases by 38% from 0.029 to 0.04, and the CHN/USA pair is less negative (from -0.016 to -0.006). These results show that the linear correlation is highly biased due to the incorrect normality assumption. One of them even changes its sign from positive to negative. Comparing these constant dependence across pairs, we can see that the highest constant dependence comes from the CHN/HKG pair, followed by the CHN/JPN and CHN/DEU pairs with all positive signs. This is reasonable since China and Hong Kong have very close economic relationship. However, all constant dependence levels are relatively low since the highest one, 0.086, is less than 0.1, which implies that Chinese financial market is not quite dependent on others. It is noticed that the signs and magnitude of dependence in Normal copula is more consistent with those of Spearman's correlation than with linear correlation in Table 1. This again verifies the argument that linear correlation is inappropriate in certain conditions.

Since the constant case can be considered as restricted version of time-varying evolution equation with two restrictions of $\alpha = 0$ and $\beta = 0$, we then perform formal likelihood ratio test to check which model is preferred. The null is that the restricted version with constant dependence of the model is not rejected as one moves to unrestricted model with time-varying dependence. According to test statistics presented in Panel A2, the null is rejected only in the CHN/HKG pair at 5% significance level, hence the time-varying model is preferred only in this pair. The constant normal copula models are preferred in all other five pairs at 5% significance level. We should conclude that the time path of dependence in the CHN/HKG pair derived from time-varying model would be more informative than others. However, given the fact that the null can be rejected at 10% significance level in the CHN/DEU and CHN/GBR pairs, the time-varying models of these two pairs could potentially provide some insights on the changes of dependence over time. The dynamics of dependence are captured by coefficients in evolution equations. The time path of dependence parameters are presented in Figure 2-7. It can be seen that most of time paths are close to white noise, but the CHN/GBR and CHN/HKG pairs seem to be informative (see Figure 4 and 5). This is shown in the estimates of evolution equation as the persistence coefficient $\beta's$ are relatively high compared to variation coefficients $\alpha's$, which means that the time-variation effects dominate in these two pairs. In Figure 1, for the CHN/DEU pair, the dependence is very volatile over time and reaches an extremely high level on March 2007. In Figure 4, for the CHN/GBR pair, the time path of dependence is relatively clear. It is clear that the dependence was increasing throughout last three month during 2007. In Figure 5, for the CHN/HKG pair, we don't find dramatic change in dependence level on July 1997 when Hong Kong left British rule though the dependence went up a little bit after July 1st, 1997. One explanation would be this event was well-predicted long time back, hence this is not considered as shock even if there was still some downturn in Hong Kong market. And we don't find significant increase in dependence level during 1997 and 1998 when Asia financial crisis was present, at least the highest peak within this period is not as high as that during last year. This is because China is relatively independent of other financial markets in Asia like Hong Kong, hence the CHN/HKG dependence didn't change much during that crashing period. Interestingly, we cannot find any significant change for all pairs in December 2002 when A share was initially open to qualified foreign institutional investors (QFII). In 2007, the dependence was increasing in general. The CHN/USA pair doesn't exhibit informative time path though it reaches a extreme peak in March 2007.

SJC Copula According to Table 5 Panel B1, in the constant tail dependence case, most of upper and lower tail dependence are close to zero except the CHN/HKG pair. This indicates that China and Hong Kong exhibits some degree dependence of extreme events in stock markets. In particular, lower tail dependence is slightly higher than upper one, hence there is higher probability of joint extreme events during downturn period than during upturn period. This is also true for the CHN/JPN pair, even though the magnitude of tail dependence is less than that of the CHN/HKG pair. For other pairs, there is no observable tail dependence, hence the joint extreme events will be less likely to happen in these pairs. Therefore, China is not affected by the extreme events in western stock market in general. Put differently, if western stock markets experience extreme downturns or upturns, then we shouldn't expect it would happen to China at the same time.

In Figure 5, for the CHN/HKG pair, even if the constant upper tail dependence is smaller than lower tail dependence, the time path of upper tail dependence is more informative than that of lower tail dependence. We can see that there exists several peaks with the highest one approaching 0.3. This shows that the time-varying model can give us further insights on the the change in dependence structure throughout the sample period. There is no strong evidence of asymmetric tail dependence in all pairs except the CHN/HKG pair, in which the lower tail dependence is 1.5 times upper tail dependence. We also conduct likelihood ratio tests with four restrictions since we have two separate evolution equations for τ^U and τ^L. The results can be found in Table 5 Panel B2. It turns out that the time-varying models are preferred in the CHN/DEU, CHN/FRA, CHN/GBR and CHN/USA pairs while the constant models are better in the CHN/HKG and CHN/JPN pairs.

[Table 5]

4.3.2 Results for U.S.-related Copula Models

Table 6 reports US-related Normal and Symmetrized Joe-Clayton (SJC) copula parameter estimates for both constant and time-varying cases.

Normal Copula In Table 6 Panel A1, we find that dependence estimates are revised by Normal copula compared to linear correlations. Specifically, the USA/DEU pair decreases by 17% from 0.455 to 0.378; the USA/FRA pair decreases by 9% from 0.428 to 0.391; the USA/GBR pair decreases by 4% from 0.413 to 0.396. However, the USA/HKG pair increases by 6% from 0.110 to 0.117 and the USA/JPN pair increases by 8% from 0.109 to 0.118. Just like China-related pairs, this revisions again show that the linear correlations are biased in non-normal situation. Most constant dependence estimates are closer to the Spearman's correlations than linear correlations. Another interesting finding is that the constant dependences are quite close to 0.39 in the first three pairs and close to 0.12 in next two pairs. There may be a general level of dependence within certain group of countries.

Similarly, we then implement likelihood ratio test to compare constant and time-varying models. The time-varying models are preferred in the USA/DEU and USA/FRA pairs, given that the null hypotheses are strongly rejected at 5% significance level. For other pairs, the constant models are preferred. Taking a look at coefficients in time-varying equations, the persistence coefficient $\beta' s$ are significantly higher than variation coefficients $\alpha' s$ in the USA/DEU and USA/FRA pairs. So persistence effects dominate. In Figure 2-3, the USA/DEU and the USA/FRA pairs show very clear and similar time-varying paths with significantly increasing dependence in the long run while others do not exhibit this pattern. In the USA/DEU pair, the dependence goes down until November 1993 and goes up thereafter. After September 1997, the dependence is consistently above the constant level at 0.378 with few exceptions and exhibits more volatile pattern. The time-varying dependence reaches as low as 0 and as high as 0.6. Interestingly, on and shortly after September 11st 2001, there exists some increase in dependence but not as dramatic as we initially expected. Compared to the USA/DEU pair, the USA/FRA pair displays similar pattern but smoother time path of dependence. The path reaches two troughs in August 1994 and July 1996 and was gradually increasing during last two years. The time-varying dependence ranges from 0.34 to 0.46. This interval is 0.12 and hence less than the interval 0.6 of the USA/DEU pair. This smaller range of the USA/FRA pair shows more stable dependence structure than that of the USA/DEU pair. Beginning from October 1996, the dependence is consistently above its constant level 0.391 with no single exception and becomes even more stable than before considering that the range further reduces to 0.06 (from 0.4

to 0.46). Moreover, the time path of the USA/FRA pair is less volatile than the USA/DEU pair. In the USA/GBR pair, not like what we expected, the time path is not quite informative and moves around the constant level 0.396. It ranges from -0.15 to 0.15 and reaches the highest peak in March 2007. In the USA/HKG pair, the dependence ranges from 0.06 to 0.2. It also exhibits large variations in relatively short period (within one year). In the USA/JPN pair, the time path is the most volatile one in U.S.-related pairs, ranging from -0.02 to 0.25. This time path is less informative than others.

SJC Copula According to Panel B, in constant case, upper and lower tail dependence are slightly different in levels for the USA/DEU, USA/FRA and USA/GBR pairs. Specifically, in these three pairs, the lower tail dependence are higher than upper tail dependence by 0.016, 0.03, 0.043, respectively. This implies that the limiting probability of U.S. stock market crashing, given that German stock market has crashed, is about 8% greater than the corresponding probability of market booming, meaning that the stock market is more dependent during market downturn than during market upturn. These findings are consistent with previous research; see Longin and Solnik (2001), Patton (2004) among others. In the USA/FRA and USA/GBR pairs, the probabilities of crashing are about 15% and 23% greater than that of booming, respectively. Therefore, the USA/GBR pair has the most asymmetric tail dependence, followed by the USA/FRA pair, and the USA/DEU pair is less asymmetric. Surprisingly, in USA/HKG pair, the upper tail dependence is 112 times lower tail dependence, meaning that the probability of U.S. market upturn, given Hong Kong market upturn, is about 112 times the corresponding probability of market downturn. This implies that the USA/HKG pair is more dependent during market upturn than during market downturn, which, to our best knowledge, has not been reported in the literature. In the USA/JPN pair, lower tail dependence is present while upper tail dependence is very small. The lower tail dependence is about 40 times upper tail dependence, showing very strong asymmetry, meaning the probability of U.S. market crashing, given Japanese market has crashed, is 40 times the corresponding probability of market booming.

For comparison purpose, we perform the likelihood ratio test with four restrictions. It turns out that all time-varying models are strongly preferred except the USA/HKG pair. In general, the evolutions of time-varying dependence parameters follow different patterns for upper and lower tail dependences. In Figure 2, for USA/DEU pair, the time path of lower tail dependence is informative but that of upper tail dependence is quite noisy. In the plot of lower tail dependence, we find that the time path of lower tail dependence is closer to its constant level before August 2000 than thereafter. After August 2000, there are five significant peaks. In particular, there is a significant adjustment period for the lower tail dependence from December 2002 though August 2003 when it goes up first and goes back to its constant level. We have similar findings for the USA/FRA pair (see Figure 3). Namely, the time path of lower tail dependence seems to be informative and very volatile while the time path of upper tail dependence is close to white noise. In the time path of lower tail dependence, we also find that there are more deviations from constant level after August 2000 than before. Interestingly, there are four significant peaks after August 2000, among which there are three peaks happened at the same periods as those in the lower tail dependence of USA/DEU pair. These two pairs exhibit similar patterns of lower tail dependence, meaning that the downturn in U.S. stock market may have similar effects on German and French stock markets in terms of probability. Also, it is clear that the lower tail dependences are relatively high in several periods in these two pairs, including 911 event in 2001, but interestingly it is not the highest peak in dependence path for each pair. In the USA/GBR pair, the time paths of lower and upper tail dependences display similar patterns, showing symmetric property. There is no significant change in both upper and lower tail dependence. In the USA/HKG pair (see Figure 5 and 6), lower tail dependence is very close to zero and upper tail dependence is move around its constant level. In the USA/JPN pair, upper tail dependence is volatile with three extreme peaks in September 1992, August and October 2005, respectively.

[Table 6]

[Figure 2-7]

4.3.3 Comparative Analysis of Dependence Structures of China and U.S.

Firstly, in general, Chinese financial market is not quite dependent upon other financial markets according to both general dependence and tail dependence. The fact that most of tail dependence parameters are close to zero implies that there is very low possibility that the joint extreme events will happen in China, given one extreme event in another country. However, U.S. market is much more correlated with other countries compared to Chinese market. This is not surprising since the index we are using is from A share market denominated in Chinese yuan which is not allowed to be traded by foreign investors until 2002.[6] After 2002, only qualified foreign institutional investors (QFII) are permitted to trade in A share market. Consequently, although there is more and more trade flows between China and western countries, the financial market in China is relatively closed to international financial market. However, we should expect that the dependence will increase in the future since it will become more and more open to foreign investors. Moreover, the western markets are all developed economies whereas China is thought of as an "emerging" market, hence portfolio managers tend to think of emerging markets as a separate asset class to invest, which may lead to this low dependence. Another interesting finding

[6] There is another B share market denominated in U.S. dollars. We don't use the index from B share market since B share is not a good representative of Chinese financial market. for two reasons: 1) it is very small in terms of market value compared to A share market and 2) domestic residents in mainland China is not allowed to invest in B share market until 2001.

is Hong Kong market, which traditionally has closer economic relationship with mainland China, has higher dependence with U.S. market than with Chinese market. One explanation would be that western portfolio managers consider Hong Kong to be "investable" over the entire sample period and feel more comfortable getting their exposure to the Chinese economy through HKG rather than investing directly in China for a long time period in our sample.

Secondly, in Figure 2 Normal case, the dependence is increasing in long term for the USA/DEU pair, whereas dependence in the CHN/DEU pair is close to white noise with an exception of a significant peak in early 2007. In Figure 3 Normal case, the USA/FRA pair shows very clear dependence path (similar pattern to USA/DEU), but the CHN/FRA pair shows just noise. In Figure 4-6, both China-related and U.S.-related pairs show very volatile dependence without clear path. In Figure 7, the USA/CHN pair shows very low dependence level with volatile time path. Therefore, there is no much comovement between Chinese and U.S. stock markets. To sum up, the time paths of dependence in USA/DEU and USA/FRA pairs are clearer and smoother than those in China-related pairs. So U.S. market comovement with these two countries will be more traceable than China. In contrast, in Britain, Hong Kong, and Japan-related pairs, China exhibits clearer and smoother time paths of dependence. In general, one may expect that the closer is the economic relationship between two countries, the clearer and more traceable time path. This clear time path will be useful in forecasting future dependence structure.

Finally, Table 7 reports results of model comparison, we can see that in Panel A, for the China-related pairs, constant models dominate in normal copula while constant and time-varying models are preferred for three pairs each in SJC copula. In Panel B, for the U.S.-related pairs, constant models got four checks in normal copula while time-varying SJC copula models got five checks out of six. It seems that, in general, constant models dominate in China-related pairs whereas time-varying models dominate in U.S.-related pairs. However, strictly speaking, model preference varies across different pairs and there is no general preference on model selection between constant and time-varying models. This implies that we have to analyze dependence structure case by case. There is no stylized preference in copula models.

[Table 7]

5 Concluding Remarks

Dependence structure is an important issue in financial contagion. The most widely used linear correlation, though provides easy and convenient way to describe comovement between two random variables, is not an appropriate dependence measure and may be highly biased in certain non-normal situations. The multivariate distribution with complex dynamic features makes linear correlation be an improper measure. In addition, asymmetric dependence in equity markets and foreign exchange markets is also documented in recent papers, such as Longin and Solnik (2001), Ang and Chen (2002), Patton (2006a) and Rodriguez (2007). These features can be easily captures in copula models with tail dependence parameters. Therefore, copula is a powerful and attractive tool to analyze dependence between margins since it doesn't requires normality in margins. Recently, the copula theory has been extended to time-varying conditional copula model by Patton (2006a), which allows for conditioning variable and the dependence parameter to vary over time. This nice model provides more insights on the dynamics of dependence structure, which makes us better understand the change in dependence structure.

In this research we use time-varying conditional copula model to study dependence structure of Chinese and U.S. stock markets. In order to use copula, we need to correctly model marginal distribution for each series. The standard $AR(p)-GARCH(1,1)-t$ model is employed to estimate these conditional marginal distributions. The test suggested by Diebold, Gunther and Tay (1998) is implemented to examine the model misspecification of conditional marginal distributions. After that, two different copulas are considered: Normal copula with regular dependence and Symmetrized Joe-Clayton copula with upper and lower tail dependence. Moreover, the dependence parameters are allowed to vary over time and $ARMA$-type evolution equations are proposed for each dependence parameter. The time paths of dependence structures for each pair are showed and analyzed. The following conclusions and implications can be reached:

Firstly, we have very fruitful empirical findings on dependence structures of Chinese and U.S. stock markets. 1) Due to the low level of dependence between Chinese and U.S. financial markets, the downturn of financial market in the U.S. will less likely affect Chinese stock market than other countries. Also, given the low tail dependence, some extreme events will not influence Chinese financial market either. This is also true for the effects of other western countries (for example, Germany, France, Britain, and also Japan) on China. However, Hong Kong has some impact on Chinese market at both general and tail dependence levels. 2) U.S. stock market is closely associated with European markets, such as Germany, France and Britain, in terms of general dependence and tail dependence. This means there is high probability that the downside in U.S. financial market and the downside in other European markets will happen simultaneously. Hence we would expect strong comovement in Europe during U.S. recession and downturn in financial market. An interesting finding is that the USA/HKG and USA/JPN pairs display similar dependence patterns in terms of general dependence, upper and lower tail dependence. In addition, we find that there may be a general level of dependence (say 0.39 for the USA/DEU, USA/FRA, USA/GBR pairs and 0.118 for the USA/HKG and USA/JPN pairs) among financial markets in developed countries and the dependence among western financial markets have a more groupwise flavor. (i.e. the USA/DEU and USA/FRA pairs demonstrate similar patterns of dependence.) 3) Compared to U.S. stock market, Chinese market is relatively independent of other major financial markets, except Hong Kong. This suggests that we should consider Chinese market as a good candidate in portfolio management to reduce risk. This model can be used in conditional asset allocation and Value-at-Risk contexts in a non-normal financial world. Conditional tail dependence may provide some useful information for portfolio weighting hence reduce exposure to downside risk. It also provide some insights on international portfolio management for global hedge fund. It is still hard to test the misspecification of the evolution equation of copula dependence parameter. We leave this topic for further research.

Secondly, time-varying model provides very important information on the time path of dependence. It shows us that the dependence could be quite volatile and deviates from it constant level frequently, hence the constant model may not be correctly describe the change in dependence. Notwithstanding the fact that time-varying model is, loosely speaking, more informative than constant model in terms of explaining the change in dependence, time-varying model doesn't always perform better than constant model. In some situations, the constant model is adequate enough to fully disclose the dependence structure, such as USA/HKG pair in our research.

Thirdly, the asymmetric behavior in dependence structure doesn't mean that there is always higher dependence during bear market than during bull market. It could be the other way around. In this paper, we find that there is higher dependence during bull market than during bear market in the USA/HKG pair. This finding, to our best knowledge, has not been documented in previous research.

Finally, regarding further research, an interesting finding is that the longer is the physical distance, the lower the dependence, at least in China-related pairs. This is in spirit similar to the intuition in gravity model of trade. Also, how can the time-varying copula model add values to VaR calculation in contrast to constant copula model will be very interesting. Moreover, one could also include time dummies to check whether or not dependence level has significant changed before and after some significant events, such as, 911 event in 2001, Hong Kong's leaving British rule in 1997. Lastly, out-of-sample performance of copula models would be an interesting topic to work on. In addition, one can employ Monte Carlo simulation method to examine the sensitivity of dependence estimates to different copula models. We leave all these topics for further research.

References

[1] Ang, A. & Chen, J. (2002), 'Asymmetric correlations of equity portfolios', Journal of Financial Economics 63(3), 443-94.

[2] Bartram, S. M. and Dufey, G. (2001). "International portfolio investment: theory, evidence, and institutional framework," Financial Markets, Institutions & Instruments 10 (3), 85-155

[3] Bollerslev, T. (1987), 'A conditional heteroskedastic time series model for speculative prices and rates of return', Review of Economics and Statistics 69, 542-547.

[4] Bollerslev, T. (1990), 'Modelling the coherence in short-run nominal exchange rates: A multivariate generalized arch model', Review of Economics and Statistics 72(3), 498-505.

[5] Fisher, R. A. (1932), Statistical Methods for Research Workers (Edinburgh: Oliver and Boyd).

[6] Hu, L. (2006), "Dependence patterns across financial markets: a mixed copula approach", Applied Financial Economics 16, 717-729

[7] Joe, H. (1997), Multivariate Models and Dependence Concepts. London: Chapman & Hall.

[8] Joe, H., Xu, J.J., (1996). "The estimation method of inference functions for margins for multivariate models," Technical Report 166, Department of Statistics, University of British Columbia.

[9] Jondeau, E. and Rockinger, M. (2006), "The Copula-GARCH model of conditional dependencies: An international stock market application," Journal of International Money and Finance 25 827-853.

[10] Kim, Y. (2005). "Dependence structure in international financial markets: evidence from Asian stock markets," UCSD Working paper

[11] Longin, F. & Solnik, B. (1995), 'Is the correlation in international equity returns constant:1960-1990?', Journal of International Money and Finance 14(1), 3-26.

[12] Longin, F., Solnik, B., (2001), "Extreme correlation of international equity markets," Journal of Finance 56 (2), 649-676.

[13] Nelsen, R. B. (2006), An Introduction to Copulas. 2nd edition. New York: Springer.

[14] Okimoto, T. (2007), 'New evidence of asymmetric dependence structures in international equity markets', Journal of Financial and Quantitative Analysis, forthcoming.

[15] Rodriguez, J. (2007), 'Measuring financial contagion: A copula approach', Journal of Empirical Finance 14, 401-423.

[16] Rosenblatt, M. (1952), "Remarks on a Multivariate Transformation," The Annals of Mathematical Statistics 23 470–72.

[17] Patton, A. J., (2002), "Applications of Copula Theory in Financial Econometrics," Unpublished Ph.D. dissertation, University of California, San Diego.

[18] Patton, A.J., (2006a), "Modelling Asymmetric Exchange Rate Dependence", International Economics Review 47 (2), 527-556

[19] Patton, A.J., (2006b), "Estimation of copula models for time series of possibly different lengths," Journal of Applied Econometrics 21 (2), 147-173.

[20] Shih, J.H., Louis, T.A., (1995). "Inferences on the association parameter in copula models for bivariate survival data," Biometrics 51, 1384-1399.

[21] Sklar, A. (1959), "Fonctions de repartition 'a n dimensions et leurs marges," Publications de l' Institut Statistique de l'Universite de Paris 8 , 229–31.

Table 1 Summary Statistics on Daily Returns

Panel A

	CHN	DEU	FRA	GBR	HKG	JPN	USA
Mean	0.084	0.040	0.030	0.025	0.050	-0.010	0.034
Std. Dev.	2.548	1.365	1.281	1.008	1.557	1.377	0.978
Skewness	6.051	-0.299	-0.118	-0.143	-0.032	0.047	-0.121
Kurtosis	159.462	7.504	6.136	6.324	13.157	5.457	7.204

Panel B

Jarque-Bera Stat.	4548766.627***	3813.035***	1826.877***	2055.549***	19054.878***	1116.843***	3275.759***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
ARCH LM Stat. (1)	2.036	181.129***	133.535***	226.405***	547.448***	42.367***	172.264***
	(0.154)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
ARCH LM Stat. (5)	19.546***	641.582***	511.811***	719.364***	762.275***	208.2944***	390.054***
	(0.002)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
ARCH LM Stat. (10)	22.467**	761.826***	664.791***	828.381***	779.087***	262.870***	476.975***
	(0.013)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
QW Stat. (1)	8.853***	1.155	0.376	0.021	2.450	4.437**	2.217
	(0.003)	(0.283)	(0.540)	(0.886)	(0.118)	(0.035)	(0.137)
QW Stat. (5)	34.731***	8.414	23.000***	29.908***	43.109***	10.809	9.632
	(0.000)	(0.135)	(0.000)	(0.000)	(0.000)	(0.055)	(0.086)
QW Stat. (10)	42.805***	20.795***	31.524***	47.157***	51.434***	12.528	21.779**
	(0.000)	(0.023)	(0.001)	(0.000)	(0.000)	(0.251)	(0.016)
Number of Obs.				4433			

Panel C

Linear Corr.	**CHN**	**DEU**	**FRA**	**GBR**	**HKG**	**JPN**	**USA**
CHN	1.000						
DEU	0.007	1.000					
FRA	0.002	0.767	1.000				
GBR	-0.002	0.684	0.780	1.000			
HKG	0.051	0.319	0.290	0.305	1.000		
JPN	0.030	0.230	0.240	0.242	0.372	1.000	
USA	-0.016	0.455	0.428	0.413	0.110	0.109	1.000

Spearman Corr.	**CHN**	**DEU**	**FRA**	**GBR**	**HKG**	**JPN**	**USA**
CHN	1.000						
DEU	0.001	1.000					
FRA	-0.004	0.710	1.000				
GBR	-0.006	0.628	0.731	1.000			
HKG	0.078	0.293	0.259	0.277	1.000		
JPN	0.028	0.232	0.221	0.220	0.347	1.000	
USA	-0.009	0.367	0.373	0.371	0.108	0.117	1.000

Notes: This table presents summary statistics of each index series. The data are 100 times the log-differences of daily stock index returns. The sample period runs 17 years from January 2nd, 1991 to December 31st, 2007, yielding 4433 observations in total. Under the null hypothesis of normality, the Jarque-Bera test statistics has a Chi-square distribution with fixed degree of freedom 2. The ARCH LM test of Engle (1982) with null hypothesis of no ARCH effect is conducted using 1, 5 and 10 lags with 1, 5 and 10 degree of freedom, respectively. Tests using other number of lags give the same results. QW statistic is the Ljung-Box statistics for serial correlation, corrected for heteroskedesticity, computed at 1, 5 and 10 lags, respectively. The asterisks, (*) (**) and (***) indicate a rejection of the null hypothesis at the 1%, 5% and 10% levels, respectively. P-values are reported in parentheses in Panel B. Correlation matrix reports the simple correlations between two country index returns.

Table 2 Results for Marginal Models

	CHN	DEU	FRA	GBR	HKG	JPN	USA
Cond. Mean (α_i)	0.029***	0.085***	0.074**	0.056***	0.073***	0.016	0.071***
	(0.015)	(0.014)	(0.015)	(0.016)	(0.016)	(0.017)	(0.011)
AR(1) (β_1)	0.049***	-0.013	-0.003	0.000	0.031**	-0.040***	-0.022***
	(0.015)	(0.016)	(0.016)	(0.016)	(0.015)	(0.016)	(0.015)
AR(2) (β_2)	0.041***	0.001	-0.024*	-0.028**	-0.007		-0.036***
	(0.015)	(0.015)	(0.015)	(0.016)	(0.015)		(0.015)
AR(3) (β_3)	0.103***	-0.023*	-0.044***	-0.029**	-0.034***		-0.040***
	(0.014)	(0.015)	(0.015)	(0.015)	(0.014)		(0.015)
AR(4) (β_4)	0.058***	0.021*	-0.000	-0.007			-0.025***
	(0.014)	(0.015)	(0.015)	(0.016)			(0.014)
AR(5) (β_5)	0.047***	-0.027**	-0.046***	-0.039***			-0.030***
	(0.014)	(0.015)	(0.015)	(0.015)			(0.014)
AR(6) (β_6)	-0.009	-0.042***	-0.012	-0.033***			-0.034***
	(0.014)	(0.015)	(0.015)	(0.015)			(0.015)
AR(7) (β_7)	0.031***		-0.046***				-0.042***
	(0.013)		(0.015)				(0.015)
AR(8) (β_8)	0.030***						
	(0.013)						
AR(9) (β_9)	0.029***						
	(0.012)						
AR(10) (β_{10})	0.040***						
	(0.012)						
AR(11) (β_{11})	0.026**						
	(0.012)						
AR(12) (β_{12})	0.034***						
	(0.011)						
AR(13) (β_{13})	0.021**						
	(0.011)						
AR(14) (β_{14})	0.038***						
	(0.011)						
AR(15) (β_{15})	0.032***						
	(0.011)						
AR(16) (β_{16})	0.004						
	(0.011)						
AR(17) (β_{17})	0.023***						
	(0.011)						
Cond. Variance (a_i)	0.094***	0.014***	0.016***	0.010***	0.014***	0.019***	0.003***
	(0.017)	(0.004)	(0.004)	(0.003)	(0.004)	(0.005)	(0.001)
ARCH(1) (c_i)	0.347***	0.081***	0.065***	0.076***	0.058***	0.064***	0.050***
	(0.040)	(0.009)	(0.007)	(0.008)	(0.007)	(0.007)	(0.006)
GARCH(1) (b_i)	0.758***	0.913***	0.925***	0.915***	0.939***	0.929***	0.949***
	(0.014)	(0.009)	(0.008)	(0.009)	(0.007)	(0.008)	(0.006)

Notes: We report maximum likelihood estimates, with standard errors in parentheses, of the parameters of the marginal distribution models for each stock index return series. The asterisks, (*) (**) and (***) indicate a rejection of the null hypothesis at the 1%, 5% and 10% levels, respectively.

Table 3 Log Likelihood of CHN-related Copula Estimation with a Constant Dependence Parameter

	CHN/DEU	CHN/FRA	CHN/GBR	CHN/HKG	CHN/JPN	CHN/USA
Normal	-0.057	-0.029	-0.001	-16.435	-3.597	-0.079
Clayton	-2.785	-0.745	-1.738	-14.217	-5.655	-0.147
Rotated Clayton	0.006	0.009	0.012	-12.466	-1.936	0.006
Plackett	-0.009	-0.177	-0.004	-15.148	-1.986	-0.007
Frank	0.0001	0.0006	0.0001	-14.722	-1.954	0.0001
Gumbel	53.515	56.439	59.873	-5.772	30.789	57.727
Rotated Gumbel	32.687	46.116	40.826	-4.835	24.701	50.591
Student's t	-2.271	-0.139	-0.204	-20.208	-4.787	-1.237
Symmetrized Joe-Clayton	0.835	3.196	2.973	-18.547	-5.615	3.597

Table 4 Log Likelihood of USA-related Copula Estimation with a Constant Dependence Parameter

	USA/DEU	USA/FRA	USA/GBR	USA/HKG	USA/JPN	USA/CHN
Normal	-342.585	-368.355	-377.028	-30.613	-31.008	-0.079
Clayton	-272.720	-303.491	-310.549	-35.012	-31.383	-0.147
Rotated Clayton	-263.903	-283.530	-275.077	-13.339	-15.932	0.006
Plackett	-328.760	-363.715	-347.602	-29.162	-30.256	-0.007
Frank	-Inf	-Inf	-Inf	-28.947	-30.201	0.0001
Gumbel	-323.616	-349.007	-336.017	-8.112	-12.813	57.727
Rotated Gumbel	-325.478	-365.379	-362.617	-30.405	-26.289	50.591
Student's t	-363.524	-401.233	-392.988	-32.789	-32.697	-1.237
Symmetrized Joe-Clayton	-357.871	-391.591	-388.734	-35.926	-33.941	3.597

Table 5 Results for CHN Copula Models

	CHN/DEU	CHN/FRA	CHN/GBR	CHN/HKG	CHN/JPN	CHN/USA
Panel A1: Normal Copula with Constant Dependence Parameter						
$\bar{\rho}$	0.005	-0.004	-0.001	0.086	0.040	-0.006
Copula Likelihood	-0.057	-0.029	-0.001	-16.435	-3.597	-0.079
Panel A2: Normal Copula with Time-Varying Dependence Parameter						
Constant(ω)	0.029	-0.008	0.0002	0.011	0.046	-0.021
α	0.345	-0.013	0.044	0.045	0.115	0.153
β	-1.905	-0.014	1.750	1.844	0.835	-1.987
Copula Likelihood	-3.039	-0.037	-2.721	-22.588	-5.632	-0.639
Likelihood Ratio (2) Stat.	-5.964*	-0.016	-5.440*	-12.306***	-4.070	-1.120
	(0.051)	(0.992)	(0.066)	(0.002)	(0.131)	(0.571)
Panel B1: SJC Copula with Constant Dependence Parameter						
$\bar{\tau}^U$	0.000	0.000	0.000	0.002	0.000	0.000
$\bar{\tau}^L$	0.000	0.000	0.000	0.003	0.0003	0.000
Copula Likelihood	0.835	3.196	2.973	-18.547	-5.615	3.597
Panel B2: SJC Copula with Time-Varying Dependence Parameter						
ConstantU	-13.865	-13.865	-13.865	-9.317	-23.599	-13.865
α^U	-0.001	-0.001	-0.0007	-23.839	0.00015	-0.0007
β^U	0.0003	0.00003	0.00003	-0.011	-0.0000003	0.00004
ConstantL	-12.960	-13.864	-13.864	2.277	-13.689	-13.865
α^L	-0.001	-0.0005	-0.0004	-25	-2.443	-0.0006
β^L	-0.00002	0.00003	0.00003	-11.505	-0.007	0.00003
Copula Likelihood	8.113	12.473	12.137	-20.368	-2.542	13.540
Likelihood Ratio (4) Stat.	14.556***	18.554***	18.328***	-3.642	6.146	19.886***
	(0.006)	(0.001)	(0.001)	(0.457)	(0.189)	(0.001)

Notes: The Likelihood Ratio (p) Statistic test the null hypothesis that the restricted version (with constant dependence) of a model is not rejected as one moves from restricted model to unrestricted model (with time-varying dependence) where the parameter p is the number of restrictions under the null. So we have two restrictions in Normal copula and four restrictions in SJC copula. P-values are reported in parentheses. The asterisks, (*) (**) and (***) indicate a rejection of the null hypothesis at the 1%, 5% and 10% levels, respectively.

Table 6 Results for USA Copula Models

	USA/DEU	USA/FRA	USA/GBR	USA/HKG	USA/JPN	USA/CHN
Panel A1: Normal Copula with Constant Dependence Parameter						
$\bar{\rho}$	0.378	0.391	0.396	0.117	0.118	-0.006
Copula Likelihood	-342.585	-368.355	-377.028	-30.613	-31.008	-0.079
Panel A2: Normal Copula with Time-Varying Dependence Parameter						
Constant(ω)	-0.012	-0.101	0.677	0.033	0.513	-0.021
α	0.073	0.008	0.059	0.028	-0.231	0.153
β	2.088	2.368	0.378	1.711	-2.022	-1.987
Copula Likelihood	-381.980	-374.840	-377.575	-31.546	-32.428	-0.639
Likelihood Ratio (2) Stat.	-78.790***	-12.970***	-0.986	-1.866	-2.840	-1.120
	(0.000)	(0.002)	(0.611)	(0.393)	(0.242)	(0.571)
Panel B1: SJC Copula with Constant Dependence Parameter						
$\bar{\tau}^U$	0.191	0.201	0.188	0.045	0.0007	0.000
$\bar{\tau}^L$	0.207	0.231	0.231	0.0004	0.028	0.000
Copula Likelihood	-357.871	-391.591	-388.734	-35.926	-33.941	3.597
Panel B2: SJC Copula with Time-Varying Dependence Parameter						
ConstantU	-1.813	-1.654	0.002	-10.930	-11.332	-13.865
α^U	-1.674	-2.240	-5.250	-0.473	-1.878	-0.0007
β^U	4.126	3.900	-0.748	0.000	-0.009	0.00004
ConstantL	0.839	-0.717	0.055	-1.338	-5.741	-13.865
α^L	-9.963	-3.007	-5.029	-7.338	6.520	-0.0006
β^L	0.387	1.033	-0.144	-0.040	6.768	0.00003
Copula Likelihood	-403.619	-405.521	-399.265	-36.079	-38.832	13.540
Likelihood Ratio (4) Stat.	-91.496***	-27.86***	-21.062***	-0.306	-9.782**	19.886***
	(0.000)	(0.000)	(0.000)	(0.989)	(0.044)	(0.001)

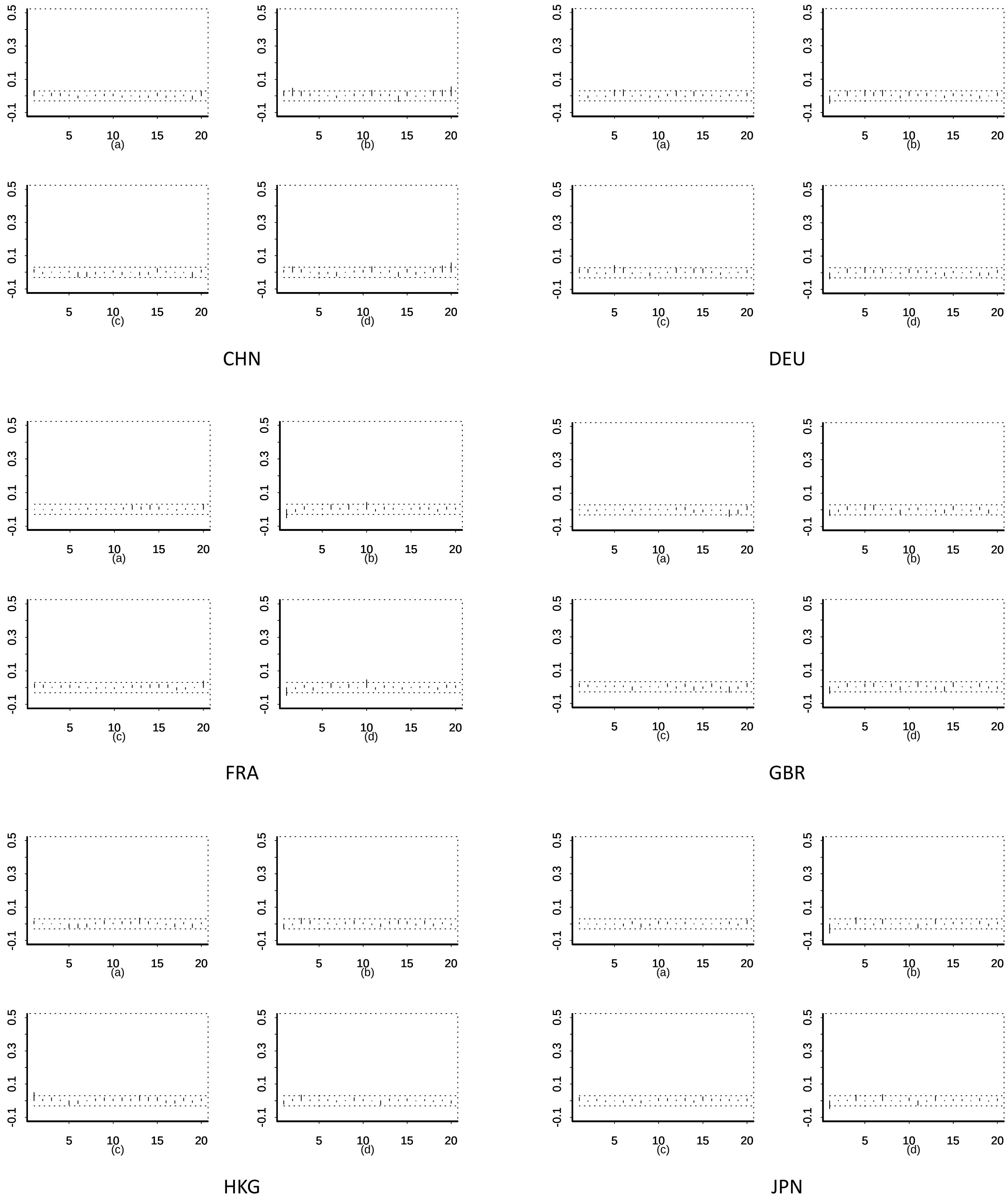
Notes: The Likelihood Ratio (p) Statistic test the null hypothesis that the restricted version (with constant dependence) of a model is not rejected as one moves from restricted model to unrestricted model (with time-varying dependence) where the parameter p is the number of restrictions under the null. So we have two restrictions in Normal copula and four restrictions in SJC copula. P-values are reported in parentheses. The asterisks, (*) (**) and (***) indicate a rejection of the null hypothesis at the 1%, 5% and 10% levels, respectively.

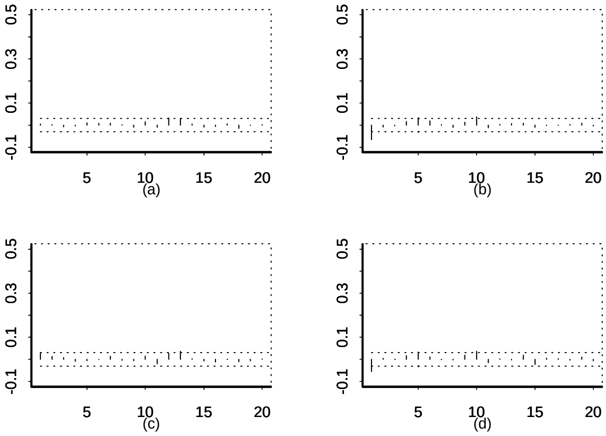
Table 7 Model Comparison: Constant vs. Time-varying Models

Panel A: China-related Models							
Model Specification		CHN/DEU	CHN/FRA	CHN/GBR	CHN/HKG	CHN/JPN	CHN/USA
Normal	***Constant***	C	C	C		C	C
	Time-varying	V*		V*	V		
SJC	***Constant***				C	C	C
	Time-varying	V	V	V			
Panel B: U.S.-related Models							
		USA/DEU	**USA/FRA**	**USA/GBR**	**USA/HKG**	**USA/JPN**	**USA/CHN**
Normal	***Constant***			C	C	C	C
	Time-varying	V	V				
SJC	***Constant***				C		
	Time-varying	V	V	V		V	V

Notes: This table is based on the results from likelihood ratio tests at 5% significance level for competing models in Table 5 and 6, where "C" means the constant model is preferred while "V" indicates the time-varying model is preferred. The asterisks, (*), indicates a rejection of the null hypothesis at the 10% level.

Figure 1 Estimates of the Autocorrelation Functions of Powers of z of AR-GARCH-t Models (Diebold-Gunther-Tay Test)

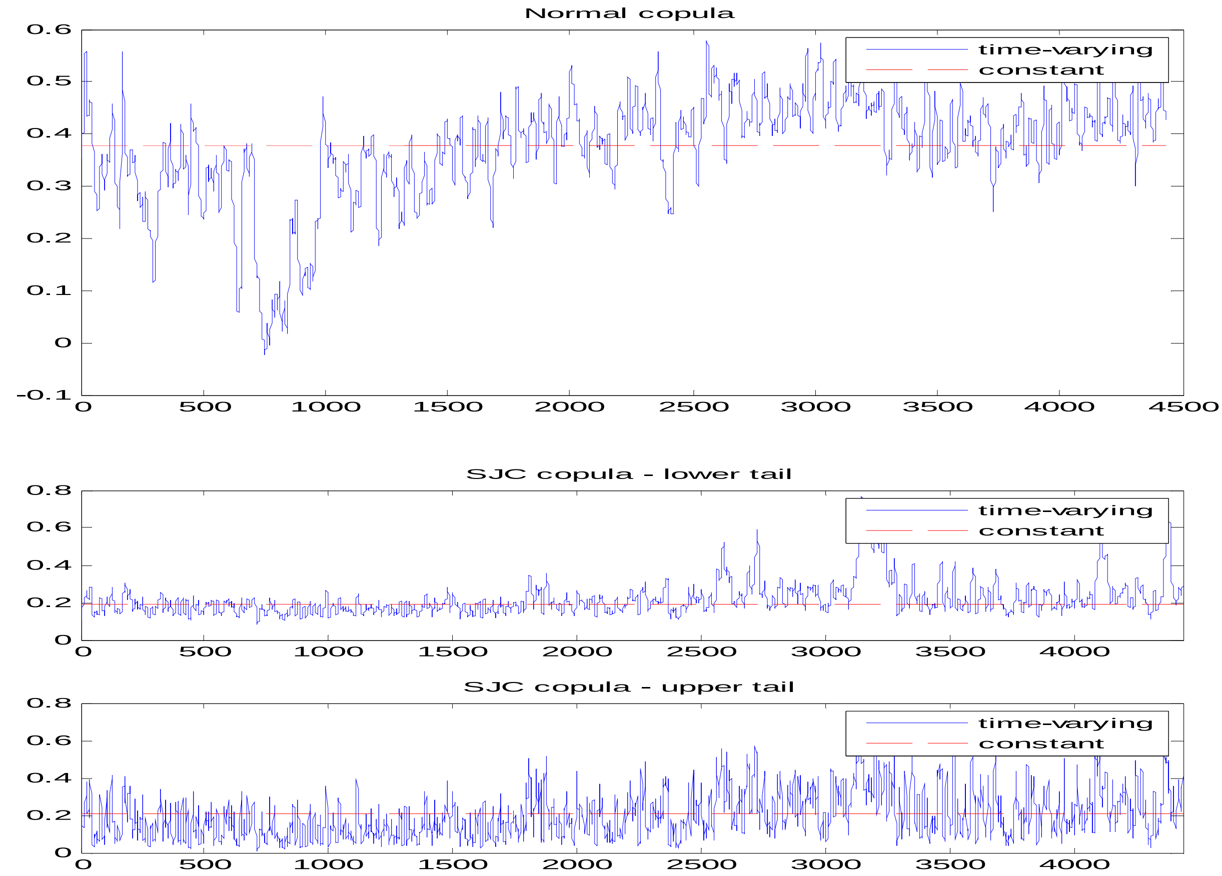




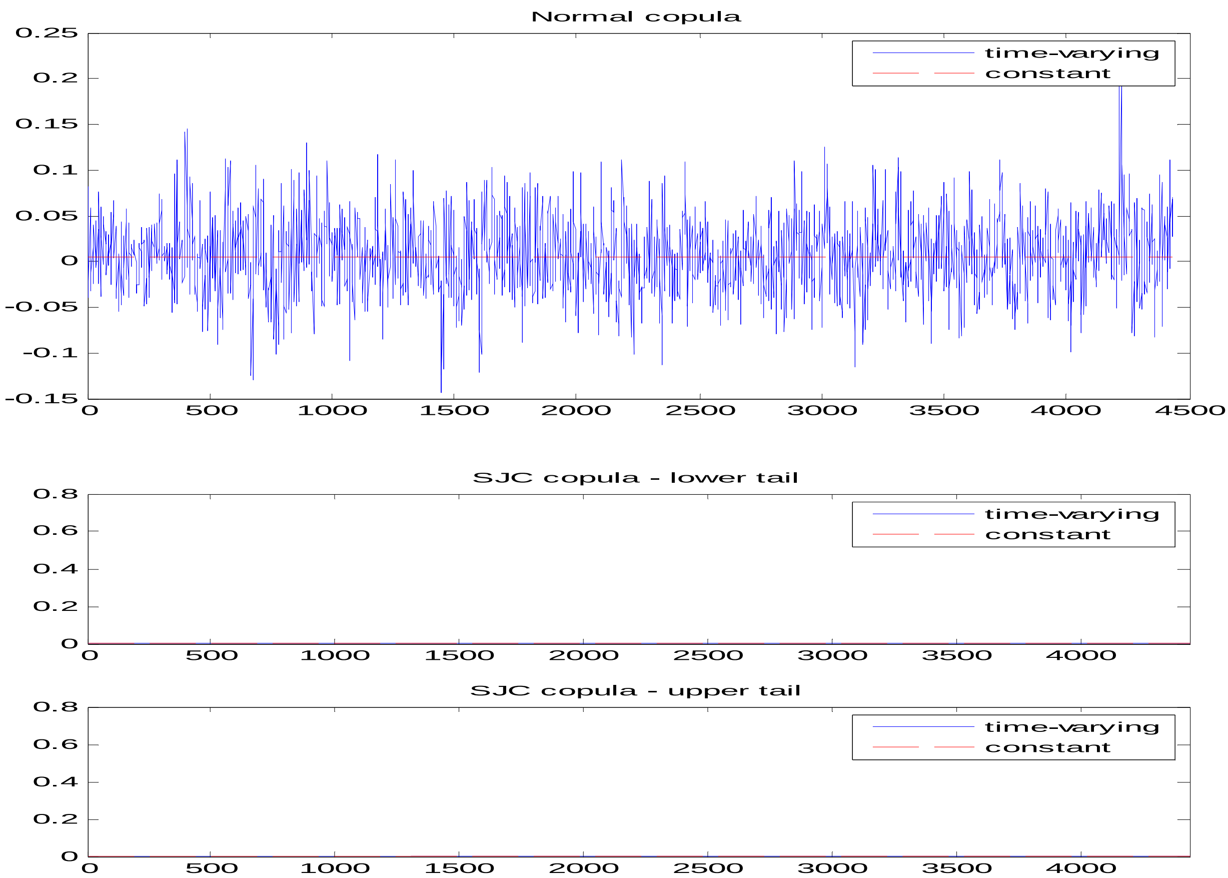
USA

Notes: z is the probability integral transform of residuals from each country's marginal model. These figures show sample autocorrelations of $(z-\bar{z})$, $(z-\bar{z})^2$, $(z-\bar{z})^3$ and $(z-\bar{z})^4$ for each country. This test is suggested by Diebold, Gunther and Tay (1998).

Figure 2 Time Path of Dependence Parameters for USA/DEU and CHN/DEU Pairs

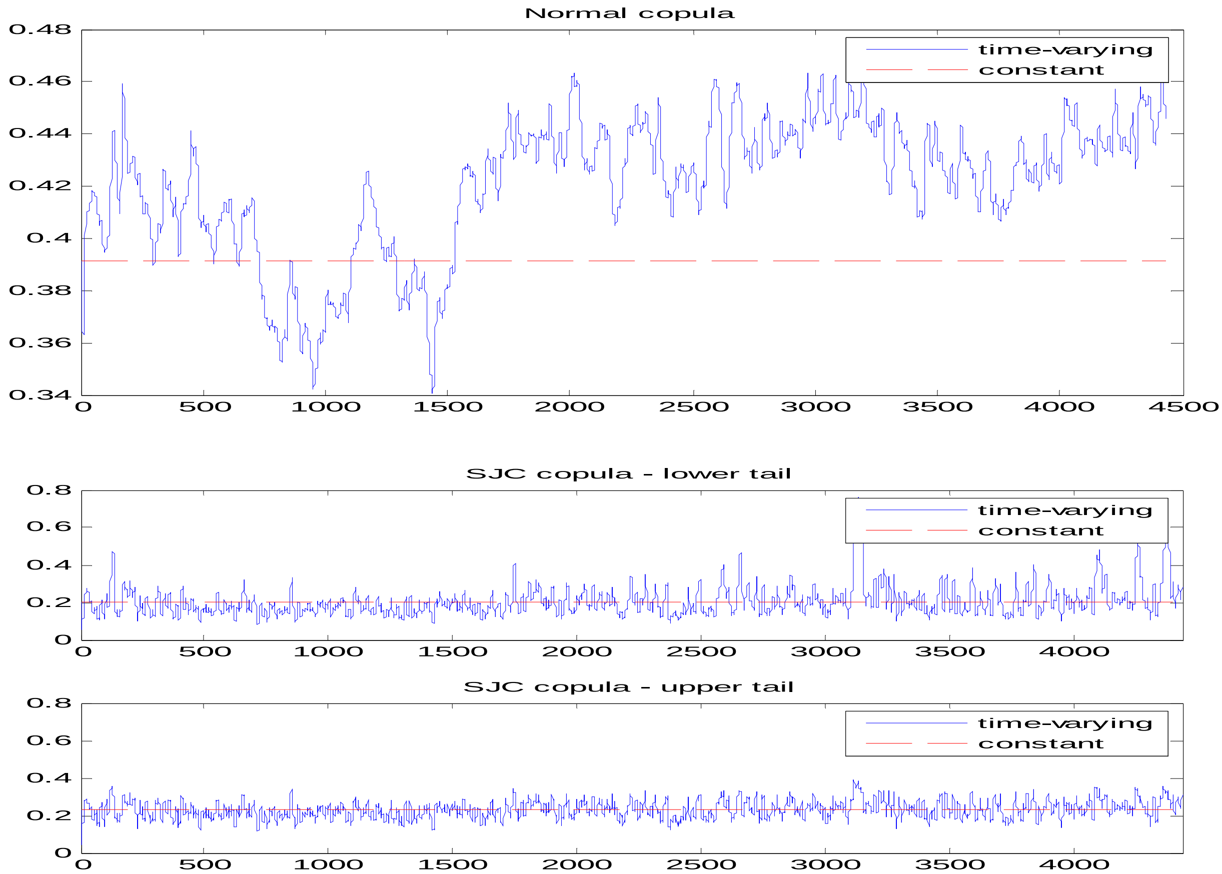


USA/DEU

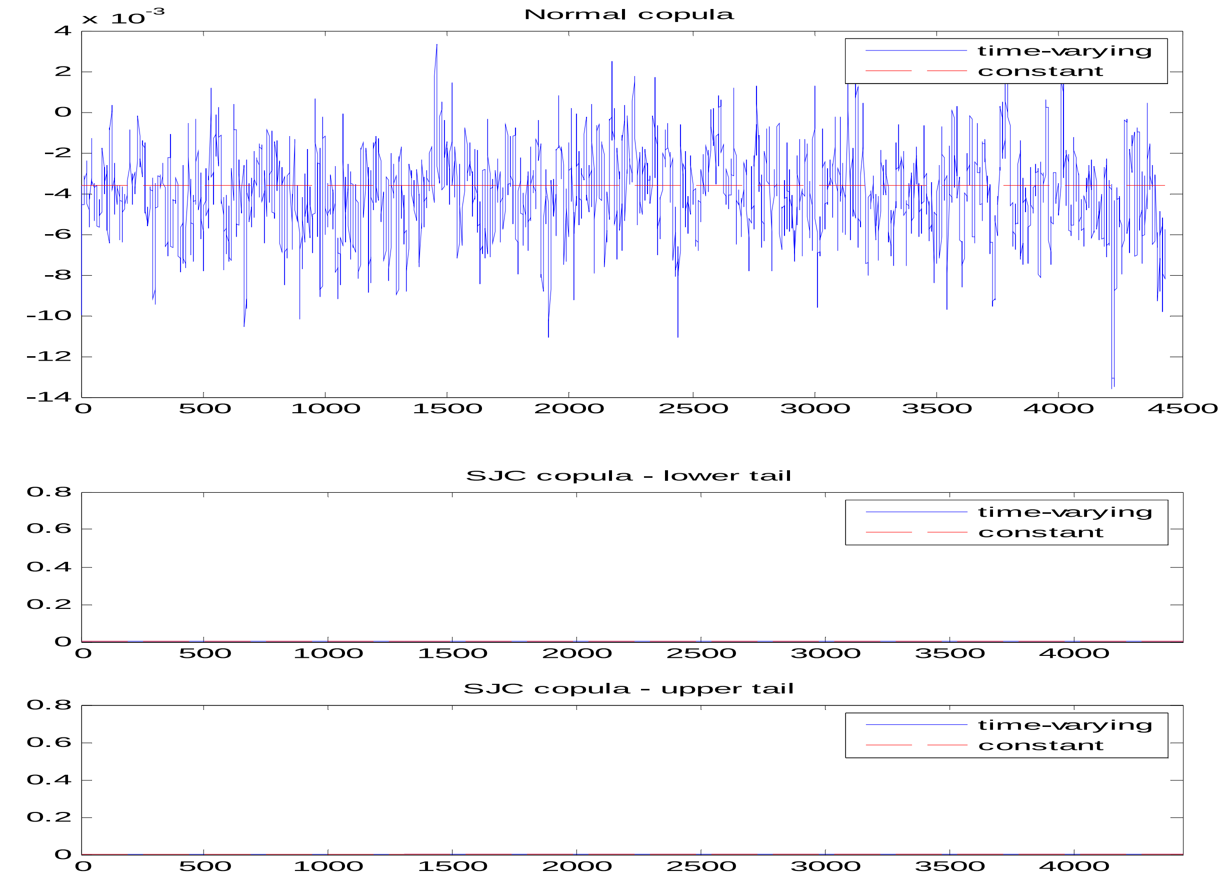


CHN/DEU

Figure 3 Time Path of Dependence Parameters for USA/FRA and CHN/FRA Pairs



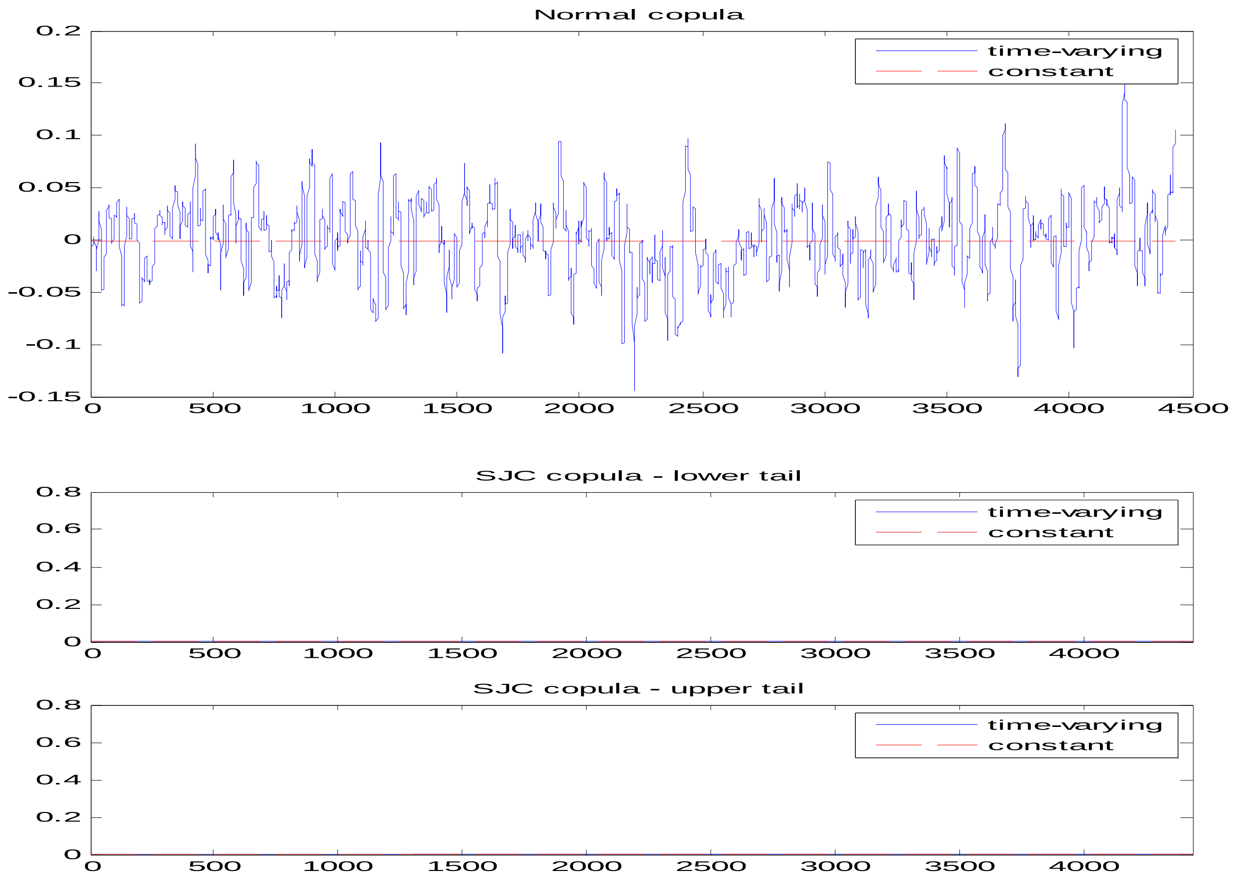
USA/FRA



CHN/FRA

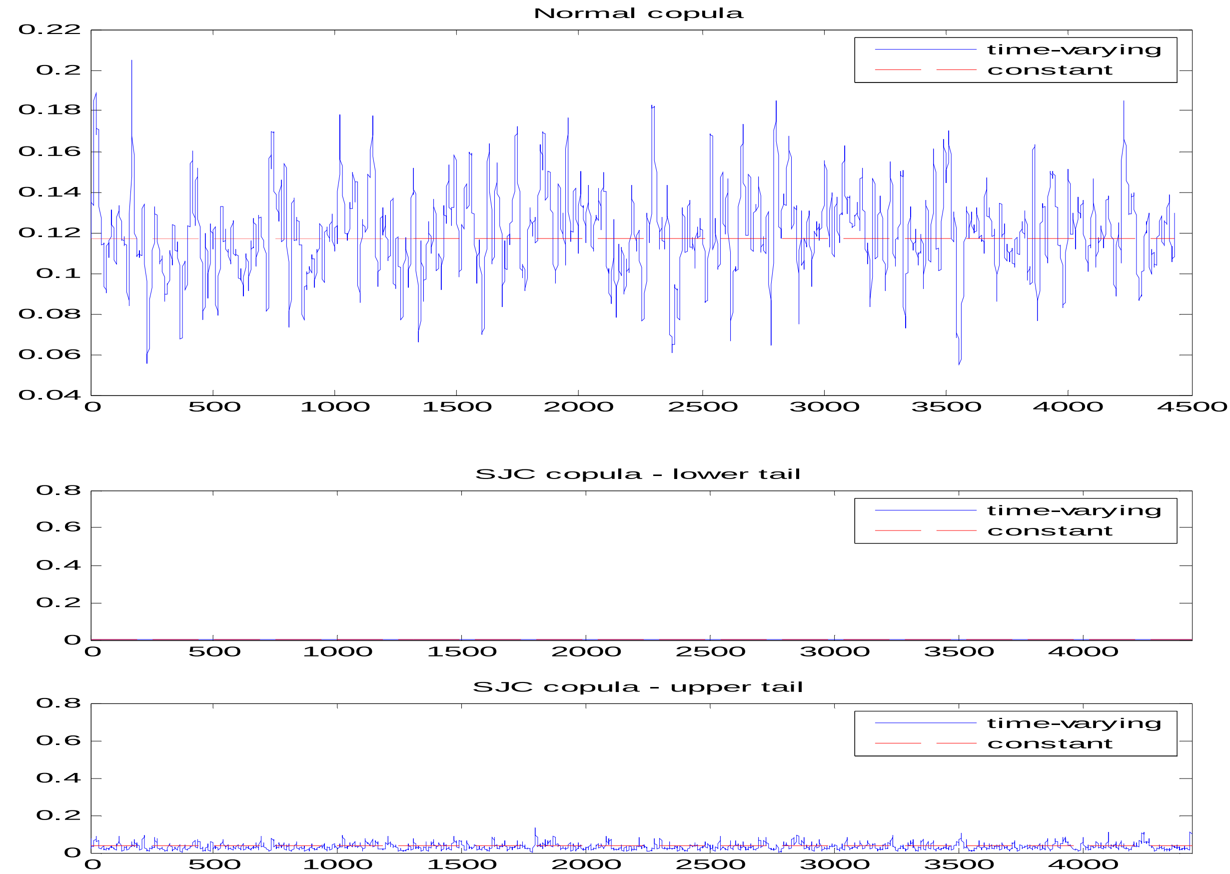
Figure 4 Time Path of Dependence Parameters for USA/GBR and CHN/GBR Pairs

USA/GBR

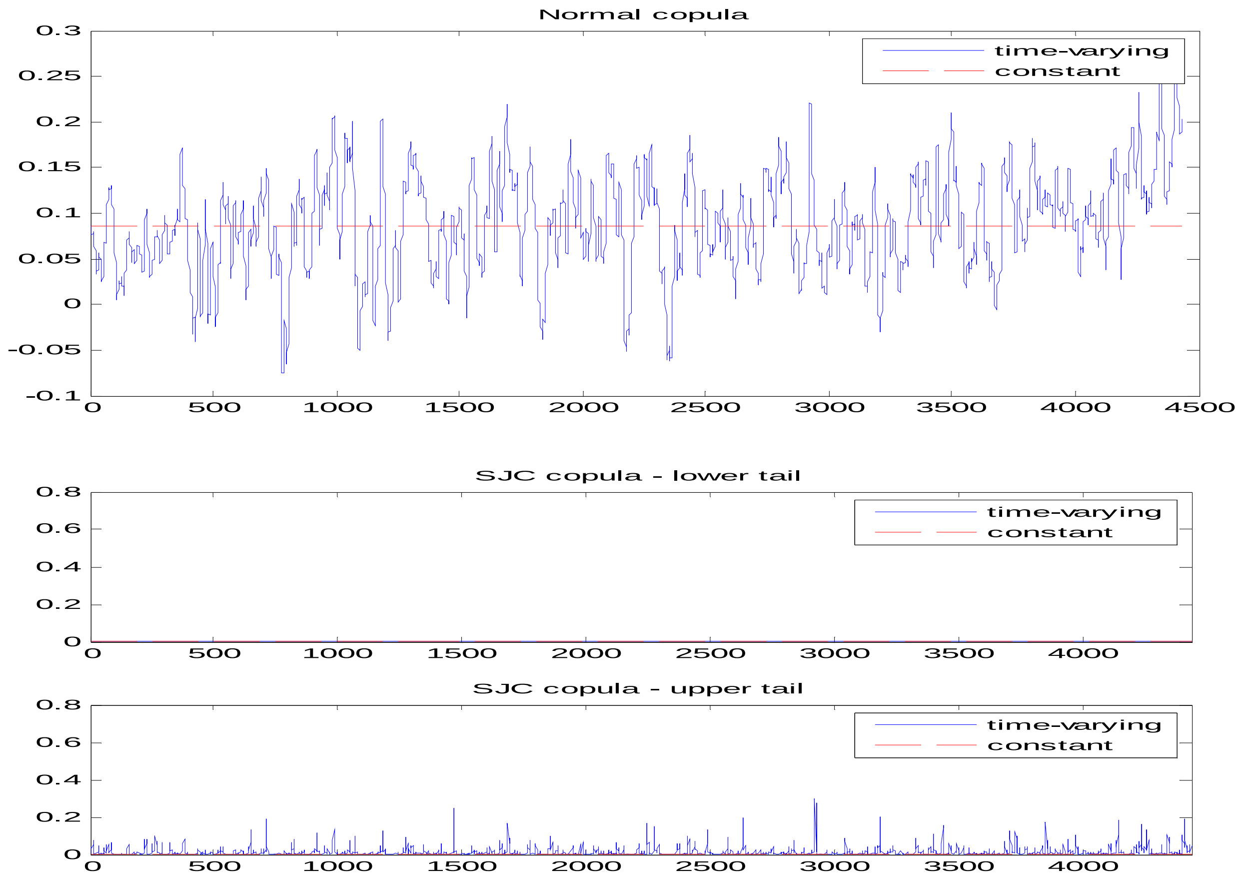


CHN/GBR

Figure 5 Time Path of Dependence Parameters for USA/HKG and CHN/HKG Pairs

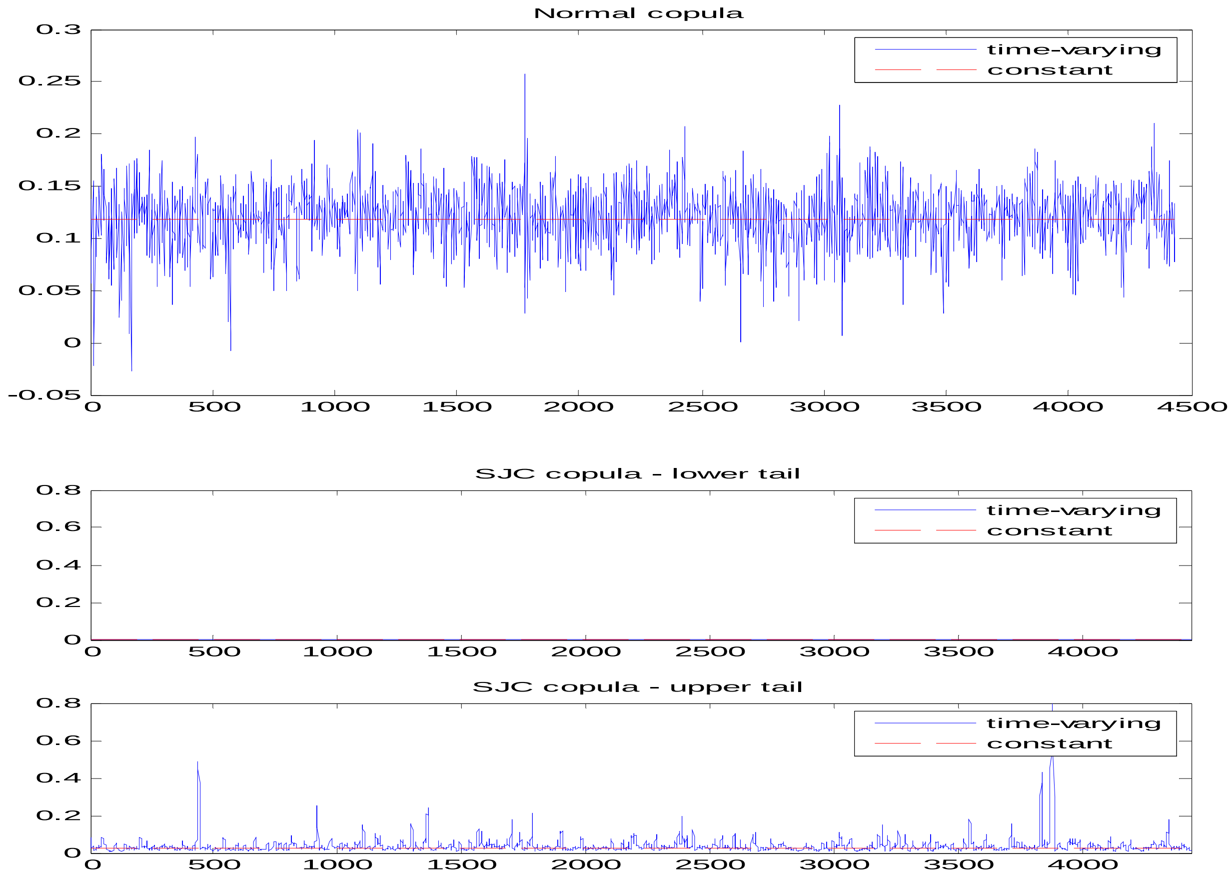


USA/HKG



CHN/HKG

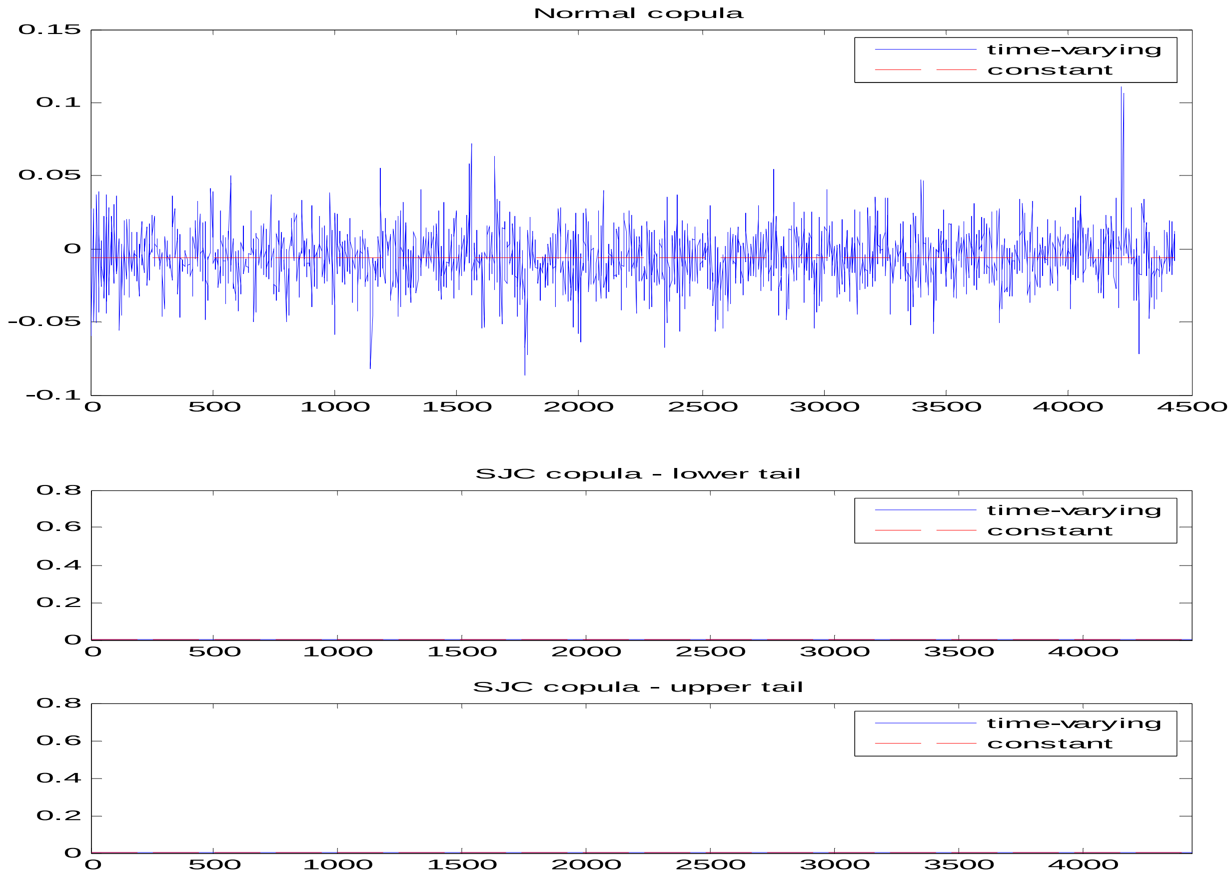
Figure 6 Time Path of Dependence Parameters for USA/JPN and CHN/JPN Pairs



USA/JPN

CHN/JPN

Figure 7 Time Path of Dependence Parameters for USA/CHN Pair



USA/CHN