# Tax Competition, Imperfect Capital Mobility and the gain from non-preferential agreements<sup>\*</sup>

Kaushal Kishore

Southern Methodist University, 3300 Dyer Street, Suite 301, Umphrey Lee Center, Dallas TX 75275-0496

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#### Abstract

The gain to competing governments from entering into binding non-preferential tax agreements (that prevents discriminatory taxation in favor of mobile capital) depends on the extent of capital mobility between jurisdictions. In particular the gain is increasing in the cost of relocation of capital and the fraction of the domestic tax base which is relatively immobile. We show this in a symmetric model of capital tax competition between two governments where all capital is imperfectly mobile and differ only in their cost of relocation.

JEL classification: F15; F21; H26; H87 Keywords; Tax competition; Capital mobility; Non-preferential regime

#### 1 Introduction

Fiscal competition between the governments of sovereign nations and other jurisdictions (including regional and local governments) is an important determinant of economic policy and the state of public finance across the world. An important concern of public economics and political economy relates to the fact that tax competition may reduce tax revenues, erode tax bases and cause other harmful effects through inefficient allocation of resources across space.<sup>1</sup> One particular aspect of this problem that has attracted considerable attention in recent years (among economists as well as policy makers) is the widespread use of preferential capital tax rates to attract mobile capital from other jurisdictions. Globalization and the removal of barriers to capital mobility has made relocation of capital easier over time and this has increased the incentives for national and other governments to engage in aggressive preferential capital taxation that can be direct (countries give

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<sup>&</sup>lt;sup>1</sup>See Wilson (1999) for a review of tax competition literature.

tax subsidy to foreign investors) or indirect<sup>2</sup> (countries set a low tax rate on investment in activities that attract large foreign capital).

In recent years, concerned by the perceived "harmful effects" of such preferential measures adopted competitively by large number of countries, several international agreements and nonbinding resolutions have been adopted by the European Union<sup>3</sup> (EU)<sup>4</sup> and Organization for Economic Co-operation and Development (OECD)<sup>5</sup> in order to impose restrictions on preferential taxation among member countries and to take joint action against continuation of preferential tax regimes by non-member countries. The primary "harmful effect" motivating such agreements appears to be the erosion of tax revenue rather than economic efficiency. However, apart from the EU and OECD, there does not appear to be any other demand for imposition of restrictions on preferential capital taxation<sup>6</sup>, despite widespread policy cooperation and coordination between blocks of nations on other issues such as trade barriers and monetary policy. It appears, therefore, that the economic gains from multilateral restriction on preferential capital taxation may not always be positive and may depend on the economic fundamentals of the competing nations. We investigate the economic foundations of non-preferential capital tax agreements. In particular, we analyze the strategic incentives and gains from such agreements in the context of capital tax competition and relate them to the composition and degree of mobility of capital bases .

This paper is a contribution to the significant theoretical literature that has focused on the comparison of tax revenues generated under capital tax competition with and without restrictions on preferential taxation. Keen (2001) analyzes a symmetric game of tax competition between two countries that compete over two exogenous capital bases and shows that if the elasticity of investment flow with respect to tax differential is not too high, then tax revenues generated in Nash equilibrium are actually higher under preferential taxation (where the two countries set discriminatory taxes on the two bases) relative to non-preferential regimes distorts tax rates (as optimal tax rates are different for capital bases with different elasticity) and spread competition for more elastic investment to less elastic investment, resulting in lower total tax revenues. Wilson (2005)

<sup>&</sup>lt;sup>2</sup>It is important to notice that a preferred treatment of foreign residents is often granted indirectly rather than directly. Ireland, for instance, levied only a 10% tax rate on corporate income in the manufacturing and financial service sectors instead of the standard rate (32%).

<sup>&</sup>lt;sup>3</sup>Main emphasis in the meeting of Council of Economics and Finance Ministers (1997) was to formalize "a design to detect such tax measures which unduly affect the location of business activity in the Community by being targeted merely at non-residents and by providing them with a more favorable tax treatment than that which is generally available in the Member State concerned. In 1998 EU Group established to identify harmful tax measures. By Nov 1999, Group identified 66 harmful tax measures".

<sup>&</sup>lt;sup>4</sup>The European Commission's (1997) "code of conduct on Business Taxation" is a non-binding resolution among member states to avoid preferential taxation of certain activities including foreign investment.

<sup>&</sup>lt;sup>5</sup>In 1998, the OECD adopted its "Guideline on Harmful Preferential Tax Regimes" (see, OECD, 1998). OECD (2000) Committee on Harmful Tax Practices identified 47 preferential tax regimes. In its progress report, OECD (2004) mentions that, 18 of these abolished, 14 amended and 13 were not found to be harmful on further analysis. OECD (2006) report states "The Committee considers that this part of the project has fully achieved its initial aims. Future work in this area will focus on monitoring any continuing and newly introduced preferential tax regimes identified by member countries".

<sup>&</sup>lt;sup>6</sup>Indeed, by the year 2007 there were only fourteen bilateral agreements for the exchange of information for tax purposes (which is absolutely essential for monitoring and identifying preferential regimes).

obtains a similar result when one of the two capital bases is perfectly mobile. These theoretical results then lead to the natural question: why do countries enter into non-preferential tax agreements? The literature identifies three factors that may explain this; "home bias", endogenous size of capital base and asymmetry.

Haupt and Peters (2005) introduce home bias in the Keen (2001) model by assuming that each country has a natural advantage in one of the two capital tax bases in the sense that when both countries set equal tax rates for that base, one country gets a larger fraction of the investment from that base. Under the preferential regime, each country is more aggressive and offers an excessive tax discount to the base in which it has a disadvantage which reduces total revenues of both countries. In contrast, when each country chooses the same tax rate for the aggregate of the two tax bases, neither has any incentive to be more aggressive than the other so that total revenues may be higher.

Janeba and Smart (2003) consider a generalized version of Keen's model where the size of the tax base is not exogenous but depends on the tax rate. Under the preferential regime, the tax rate on the more elastic tax base is lower and therefore the tax base itself is larger. Restrictions on preferential taxation increases the tax rate on the more elastic tax base (the larger base under preferential taxation) and decreases the tax rate on the less elastic capital base (which is smaller in size under preferential taxation). Therefore, starting from an unrestricted preferential regime, a small restriction on preferential taxation increases tax revenue. A complete ban on preferential taxation is desirable if the resultant increase in the tax rate on more elastic capital bases does not erode the base significantly.

If countries differ significantly in size and one of the capital bases is perfectly mobile, it has been shown that non-preferential tax regimes can generate higher tax revenue.<sup>7</sup> In a model with two tax bases (one of which is perfectly mobile and other is perfectly immobile) Janeba and Peters<sup>8</sup> (1999) argue that while the under preferential regime countries engage in Bertrand tax competition which reduces tax on mobile capital to zero, under then non-preferential regime the country with a smaller captive segment of investors (immobile capital) is more aggressive in tax competition and focuses on attracting mobile capital. Conversely the country with a larger captive segment is content with earning high tax revenue from immobile segment.<sup>9</sup> The smaller country (country with a smaller immobile capital base) earns higher tax revenue under the non-preferential regime while the tax revenue of larger country (country with a larger immobile capital base) remains exactly the same as under the preferential regime<sup>10</sup>. Emphasizing the role of asymmetry in obtaining this result, Wilson (2005) shows that when countries are symmetric, preferential and non-preferential

<sup>&</sup>lt;sup>7</sup>Unlike asymmetry in size and composition of capital bases between countries, asymmetry in productivity of capital between two countries may actually increase the appeal of preferential taxation if the smaller country is also less productive. Nicolas, Steeve and Wilson (2007) argue that when one of the capital bases is perfectly mobile and the other is perfectly immobile, non-preferential regimes cause the smaller country to be more aggressive in reducing taxes (smaller domestic immobile capital segment); and if the smaller country is also less productive, then it may lead to a reduction in the total output and hence the joint tax revenues of the competing countries.

<sup>&</sup>lt;sup>8</sup>Wang (2004) generalizes this model further, allowing for mixed strategy Nash equilibrium. His finding lends support to the main result of Janeba and Peters (1999).

<sup>&</sup>lt;sup>9</sup>This result may change if the small country is also less productive.

<sup>&</sup>lt;sup>10</sup>Also see Peralta and Ypersele (2005) for tax competition among assymptric countries.

regimes generate equal tax revenues.<sup>11</sup>

In this paper we focus on the effect of differences in the extent of capital mobility on the comparison of tax revenues generated under preferential and non-preferential regimes. In particular, we consider a framework where there is no asymmetry between the countries, the tax competition game is fully symmetric, there is no home bias (in the sense of Haupt and Peters, 2005), and the total size of the tax bases are exogenously fixed (unaffected by the tax rates). We show nonetheless that, a non-preferential tax regime may generate higher tax revenue if capital is not extremely mobile.

We analyze a symmetric model with two countries (jurisdictions) in which the governments engage in capital tax competition in order to maximize their tax revenue. Each country has a unit mass of capital owners which differ only in their capital mobility, and total investment is exogenously fixed. A fraction of capital owners in each country is perfectly immobile while the other fraction is partially mobile in the sense that investors have to incur a fixed cost of capital relocation to invest in the other country. The model generalizes the analysis in Wilson (2005) and Marceau, Mongrain, Wilson (2007) to allow perfectly mobile capital owners to be imperfectly mobile. All mobile capital owners are assumed to be identical - this allows us to treat the cost of relocation as a single parameter. The degree of capital mobility in our model is captured by the fraction of investors in each country which are mobile and the fixed cost of relocation of mobile investors.

Under preferential regimes, governments can set different tax rates for mobile and immobile investors that locate in their jurisdiction (independent of their origin). Under non-preferential regimes, each government sets the same tax rate on all investment made within its jurisdiction. We compare the Nash equilibrium tax revenue outcomes between the preferential and non-preferential regime and characterize how this comparison depends on the degree of capital mobility.

There are three main contributions of this paper. First, we provide an economic foundation on the basis of the degree of capital mobility for existing non-preferential tax agreements in Europe and OECD countries and account for the fact that there are large parts of the world where governments do not appear to be moving towards any such tax agreement. In particular, we argue that a combination of the cost of capital relocation and composition of tax bases in terms of capital mobility may provide some understanding of this phenomenon. In this respect, our analysis complements the existing literature that has focused on other factors such as asymmetry, home bias and effect of tax rate on the size of tax base. In particular, we provide a clear characterization of the comparative statics of capital mobility on the tax revenue gains from switching to a non-preferential tax agreement. Second, we characterize mixed strategy equilibrium for a class of Bertrand tax/price competition games where competitors need to undercut rivals by an exogenous discrete amount in order to steal business or attract investors from a rival jurisdiction. The existing

<sup>&</sup>lt;sup>11</sup>The introduction of asymmetry between competing jurisdictions does not necessarily overturn Keen's result on superiority of preferential tax regimes. For example, Bucovetsky and Haufer (2006) show that Keen's result holds if countries differ in size.

literature on models of homogenous good price competition with captive consumer segments<sup>12</sup> as well as tax competition with perfectly immobile segments, assumes that non-captive consumers or mobile investors always move to the firm or the country with lower price/tax<sup>13</sup> i.e., there is no fixed cost or relocation or preference for one product over another. The mixed strategy equilibrium characterized in our paper should be useful to a much larger literature in applied game theory and industrial organization. Finally, in the equilibrium of our model, though all mobile capital owners are identical and countries set different tax rates with probability one, both countries receive a positive share of investment from mobile capital owners i.e., the highly elastic tax base.<sup>14</sup>

The remainder of the paper has the following structure. Section 2 introduces the model of tax competition. Sections 3 and 4 characterize the Nash equilibrium of the tax competition game under non-preferential regime and preferential regime, respectively. Section 5 compares the tax revenues generated under the two regimes. Section 6 concludes.

#### 2 Model

We consider two identical countries/jurisdictions/governments labeled as country 1 and country 2. There is a unit mass of capital owners in each country whom we shall refer to as agents. Each agent is endowed with one unit of capital. In each country,  $\lambda$  agents are internationally mobile (though imperfectly) and  $1 - \lambda$  agents are perfectly immobile, where  $0 < \lambda < 1$ . We refer to the segment of mobile agents as the mobile capital base and that of immobile agents as the immobile capital base. One can think of perfectly immobile capital owners as being individuals whose cost of moving capital to (or investing in) the other country is higher than the total return (profit) on investment. If a mobile agent in country *i* seeks to invest in country *j* where  $i \neq j$ , he incurs an additional exogenous fixed cost  $F \geq 0$ . This reflects various mobility costs including informational costs and the uncertainty involved in accessing investment opportunity in the foreign country. Parameter *F* therefore captures barriers to the international mobility of capital. We assume that capital is taxed only in the country where it is invested (independent of the source or country of origin of the investors)<sup>15</sup>. Governments maximize tax revenue<sup>16</sup> and capital owners maximize net return on capital after tax. We assume that the return on capital (productivity of a unit of capital) is

 $<sup>^{12}</sup>$ Among many, see, example; Butters (1977), Shilony (1977), Rosenthal (1980), Varian (1980), Deneckere et al. (1992), Narasimhan (1988).

 $<sup>^{13}</sup>$ See for example see, Janeba and Peters (1999), Wang (2005), Wilson (2005) and Nicolas, Steeve and Wilson (2007) among many.

<sup>&</sup>lt;sup>14</sup>In contrast to Wilson (2005), Janeba and Peters (1999), Nicolas, Steeve and Wilson (2007)) In a CES Lecture Course on Theories of Tax Competition, Wilson emphasizes that the next step in research on multiple tax bases is, "Build a model with an equilibrium where both countries obtain some of a highly elastic base. (Does this model exist?) Are preferential or non-preferential regimes desirable?".

<sup>&</sup>lt;sup>15</sup>Taxing foreign source income can be extremely difficult and costly for governments. As noted by Janeba and Smart (2005) and also by Wilson (1999), taxing foreign-source income implies severe administrative and tax compliance problems.

<sup>&</sup>lt;sup>16</sup>This is not identical to assuming that the government is Leviathan (see, Edwards and Keen, 1996).

identical in both countries<sup>17</sup> which is normalized to 1 for simplicity. Thus investment decisions of agents are only guided by the tax rates set by the two competing governments and the cost of mobility. The maximum tax rate a country can impose on any form of capital is equal to 1.

We consider two alternative game forms depending on whether or not governments can engage in preferential taxation. The first is the preferential regime. Here the game consists of two stages. In stage 1, competing governments simultaneously announce separate tax rates on a immobile and a mobile capital base. In stage 2, mobile agents in each country decide whether to invest in country 1 or country 2 after observing tax rates set by competing countries in stage 1. It is obvious that under the preferential regime governments set the highest possible tax rate on the immobile capital base(equal to 1). Competition for the mobile capital base is similar to a Bertrand price competition with the exception that a country has to undercut the tax rate of the competing country by a strictly positive discrete amount F to attract mobile agents. Let  $\tau_p^i$  and  $\tau_p^j$  be tax rates set by governments of country i and country j, respectively, on the mobile capital base. Then a mobile agent in country i invests in country i as long as  $\left|\tau_p^i - \tau_p^j\right| < F$ , and he invests in country j if  $\tau_p^i \geq \tau_p^j + F$ . The revenue function of country i in stage 1 under the preferential regime can be described as

$$R_{p}^{i}\left(\tau_{p}^{i}, \tau_{p}^{j}\right) = (1 - \lambda) + \begin{cases} \lambda \tau_{p}^{i} & \text{if } \left|\tau_{p}^{i} - \tau_{p}^{j}\right| < F\\ 2\lambda\left(\tau_{p}^{i}\right) & \text{if } \tau_{p}^{i} \le \tau_{p}^{j} - F\\ 0 & \text{if } \tau_{p}^{i} \ge \tau_{p}^{j} + F, \end{cases}$$
(1)

where i, j = 1, 2 and  $i \neq j$ . The first term  $(1 - \lambda)$  is the tax revenue of country i from the immobile capital base and second term is the tax revenue from the mobile capital base. If  $\left|\tau_p^i - \tau_p^j\right| < F$ , mobile agents find it profitable to invest in their respective home country as difference in tax rates set by competing countries is not enough to compensate for the cost of relocating capital to the country with lower tax. Hence, country i earns the tax revenue equal to  $\lambda \tau_p^i$  from the mobile capital base. When  $\tau_p^i \leq \tau_p^j - F$ , mobile agents in country j also find it profitable to invest in country i. Thus country i receives the tax revenue amounting to  $2\lambda (\tau_p^i)$ , while country j does not receive taxes from mobile agents<sup>18</sup>. The result is opposite when  $\tau_p^i \geq \tau_p^j + F$ ; country i does not receive taxes from mobile agents since all mobile agents in country j.

Second is the non-preferential regime in which the governments pre-commit to uniform tax rates applicable to both mobile and immobile capital bases. Here, as before the game consists of two stages. In stage 1 competing governments simultaneously announce uniform tax rates. In stage 2, mobile agents make investment decisions after observing tax rates set by competing governments, while immobile agents pay the tax rate set by the government of their respective country of residence. Tax competition under the non-preferential regime is similar to a price

<sup>&</sup>lt;sup>17</sup>Departing from the standard assumption of a declining marginal product of capital is frequent in the literature and simplifies our analysis. For papers which investigate the case in which capital tends to agglomerate because of an increasing marginal product, see Baldwin and Krugman (2004), Boadway, Cuff and Marceau (2004), Kind, Knarvik and Schjelderup (2000).

<sup>&</sup>lt;sup>18</sup>Our analysis will not change if we assume that when  $\tau^i = \tau^j + F$  two countries share mobile capital base in country *j* equally.

competition between two firms for a certain fraction of consumers who are informed about both firms when both firms have identical captive segments (consumers who are only informed about one of the two firms). Similar to the case of the preferential regime a country has to undercut tax rate of the competing country by a discrete positive amount F to attract capital from the competing country. Let  $\tau_{np}^i$  and  $\tau_{np}^j$  be the tax rates set by the governments of country i and country j, respectively, in stage 1. The tax revenue of country i in stage 1 under the non-preferential regime can be described as

$$R_{np}^{i}\left(\tau_{np}^{i}, \tau_{np}^{j}\right) = (1-\lambda)\tau_{np}^{i} + \begin{cases} \lambda\tau_{np}^{i} & \text{if } \left|\tau_{np}^{i} - \tau_{np}^{j}\right| < F\\ 2\lambda\tau_{np}^{i} & \text{if } \tau_{np}^{i} \le \tau_{np}^{j} - F\\ 0 & \text{if } \tau_{np}^{i} \ge \tau_{np}^{j} + F, \end{cases}$$
(2)

where i, j = 1, 2 and  $i \neq j$ . The first term  $(1 - \lambda) \tau_{np}^{i}$  is the tax revenue of country *i* from the immobile capital base and the second term is the tax revenue from the mobile capital base.

As  $\tau_{np}^i \leq 1$ , depending on F, governments may receive a lower tax revenue from the immobile capital base under the non-preferential regime when compared to the preferential regime, in which the tax rate on the immobile capital base is fixed at 1. At the same time the immobile capital base may reduce intensity of competition for the mobile capital base which may increase the over all tax revenues of competing countries. Whether a non-preferential generates higher tax revenues compared to a preferential regime depends on which of the two effects dominates.

In this model agents completely ignore the effect of their actions (investment decision) in stage 2 on tax rates set by competing governments in stage 1. Payoffs of competing governments is fully determined in terms of the parameters ( $\lambda$  and F) of the model and actions (tax rates) competing governments choose in stage 1. Hence we model tax competition under the two regimes as a symmetric one shot simultaneous move game where the payoff functions of competing governments are given by (1) and (2), respectively, under the preferential and the non-preferential regime. Equilibrium concept we use is Nash equilibrium. We make following assumption below to insure that we have a Nash equilibrium under the non-preferential regime.

Assumption (1):

$$F \ge (1-\lambda) \left(\frac{1}{\lambda} + \sqrt{1+\frac{1}{\lambda^2}}\right)^{-1} \equiv \Upsilon(\lambda)$$
(3)

We find the Nash equilibrium of tax competition under the non-preferential regime when  $F \ge \Upsilon(\lambda)$ . If we fix  $\lambda$ , then F should be at least as large as certain critical values which satisfy (3) with equality; i.e.,  $F = \Upsilon(\lambda)$ . We explain the assumption (1) in more detail in the next section.

#### 3 Non-Preferential Regime

In this section we analyze the nature of tax competition when competing governments can not set different tax rates on the mobile and the immobile capital bases. The extreme case when the fraction of capital is perfectly mobile (F = 0) has been analyzed by Wilson (2005) and Janeba and Peters (1999). But intermediary cases in which capital is imperfectly mobile have not been analyzed.

The strategy space of country *i* is defined as  $\tau^i \in [0, 1]$ , where  $i \in \{1, 2\}$ . Let  $\tau^1$  and  $\tau^2$  be tax rates set by the respective governments of country 1 and country 2. The payoff function of country *i* (which is also the gross tax revenue of country *i*) for the strategy pair  $(\tau^i, \tau^j)$  is described as

$$R_{np}^{i}\left(\tau^{i}, \tau^{j}\right) = (1-\lambda)\tau^{i} + \begin{cases} \lambda\tau^{i} & \text{if } \left|\tau^{i}-\tau^{j}\right| < F\\ 2\lambda\tau^{i} & \text{if } \tau^{i} \le \tau^{j}-F\\ 0 & \text{if } \tau^{i} \ge \tau^{j}+F, \end{cases}$$
(4)

where  $i, j = \{1, 2\}$  and  $i \neq j$ . From (3) we can see that the revenue function of a country is discontinuous when  $|\tau^i - \tau^j| = F$ . When  $|\tau^i - \tau^j| < F$ , the difference in tax rates set by competing countries is not enough to cover the cost of capital relocation. In this case even mobile agents invest in their respective home countries. When  $\tau^i \leq \tau^j - F$ , country *i* attracts investment from mobile agents of country *j*. Country *i* receives the tax revenue equal to  $\tau^i$  from domestic agents and it also receives the tax revenue of amount  $\lambda(\tau^i)$  from mobile agents of country *j*. Hence the aggregate tax revenue of country *i* sums to  $\tau^i + \lambda(\tau^i)$ . If  $\tau^i \geq \tau^j + F$ , then the tax rate in country *i* is too high compared to country *j*. In this case all mobile agents invest in country *j*. Country *i* receives taxes from its domestic immobile capital base which total  $(1 - \lambda)\tau^i$ .

DEFINITION 1: A function  $f(S) \to \Re$  defined on a convex subset S of a real vector space is quasi-concave if  $\forall x', x'' \in S, f(x) \ge \min \{f(x'), f(x'')\} \forall x \in [x', x''].$ 

To see that the payoff function given by (3) is not quasi-concave with respect to the player's own strategy when 0 < F < 1; consider  $\tau^j = 1, 0 < F < 1$  and  $\tau^i \in [1 - F, 1]$ .  $R_{np}^i(1, 1) = 1$  and  $R_{np}^i(1 - F, 1) = (1 - F)(1 + \lambda)$ . Now consider  $\delta$ , s.t.  $F > \delta > 0$ , this implies  $1 - F + \delta \in [1, 1 - F]$ . We have  $R_{np}^i(1 - F + \delta, 1) = 1 - F + \delta$ . It is clear that  $1 - F + \delta < 1$  for  $F > \delta > 0$ . Now suppose  $(1 - F)(1 + \lambda) \leq 1$ . Because  $1 - F + \delta < (1 - F)(1 + \lambda)$  for  $\delta < (1 - F)\lambda$ , we can say that the function  $R_{np}^i(\tau^i, \tau^j)$  is not quasi-concave. By symmetry the same is true for  $R_{np}^j(\tau^i, \tau^j)$ .

A pure strategy Nash equilibrium does not exist if  $F > (1 + \frac{1}{\lambda})^{-1}$ . Each country may desire a high tax rate when the other country's tax rate is low, while also desiring a low tax rate when the other country's tax rate is high. In the appendix we explain in more detail the reason for the non-existence of a pure strategy Nash equilibrium.

Mixed strategy Nash equilibrium under such circumstance has not been analyzed. Dasgupta and Maskin (1986) provide an example where a firm can sell to all consumers by undercutting the price charged by a competing firm by a small margin, resulting in a mixed strategy Nash equilibrium with atom less convex support. Varian (1980) characterizes symmetric mixed strategy Nash equilibrium of a more general price competition model with n firms and free entry. Unlike Varian's model, the case of only two countries/firms with no entry which may allow countries/firms to earn the positive tax revenue/profit is also considered in Wilson (2005), Wang (2005) and Narasimhan (1988) among others. We analyze the case when countries/firms have to undercut the tax rate/price set by a competing country/firm by a positive discrete margin to attract mobile capital/consumers. This generalization is significant as it provides tools to study many issues of economic relevance such as labor/capital movement due to tax/environmental differences and also when countries have imperfectly mobile capital base, our current exercise. The outcome will be similar when firms compete with differentiated products while maintaining a captive segment. The limitation of this equilibrium is that the two countries have equal immobile capital bases and we do not find an equilibrium when the cost of capital mobility is very small. Characterizing mixed strategy Nash equilibrium with unequal immobile capital bases/captive segments and relaxing the constraint we impose provides further challenge.

We only consider symmetric mixed strategy Nash equilibrium. Hence we remove the superscript to identify the individual country. To find the mixed strategy Nash equilibrium, we consider G(.), the cumulative distribution function (mixed strategy) of tax rates competing countries use in an equilibrium. Subsequently, sop(dG) denotes the support of the mixed strategy Nash equilibrium. Thus for any  $\tau \in sop(dG)$ , the expected tax revenue of a competing country can be written as

$$E(R) = \tau (1 - \lambda) + \tau \lambda \left[1 - G(\tau - F)\right] + \tau \lambda \left[1 - G(\tau + F)\right] \quad \forall \tau \in sop(dG).$$
(5)

For any tax rate  $\tau$ , a country always receives taxes from its immobile capital base. With probability  $[1 - G(\tau - F)]$ , it retains the domestic mobile capital base and with probability  $[1 - G(\tau + F)]$ , it also attracts mobile agents from the competing country. In proposition (1) below we describe the outcome of the tax competition under the non-preferential regime formally. See appendix for complete description of the mixed strategy Nash equilibrium.

**Proposition (1):** If  $F \ge (1 + \frac{1}{\lambda})^{-1}$ , a unique pure strategy Nash equilibrium exists. In equilibrium, competing countries set tax rate equal to 1 and earn tax revenue equal to 1. If  $F < (1 + \frac{1}{\lambda})^{-1}$ , a pure strategy Nash equilibrium does not exist, however a mixed strategy Nash equilibrium exists.

(I) If  $\frac{\lambda}{1+\lambda+\sqrt{1+\lambda^2}} < F < (1+\frac{1}{\lambda})^{-1}$ , in a symmetric mixed strategy Nash equilibrium, the tax revenue of a competing country is equal to  $1-\Psi$ , where

$$\Psi = \frac{1}{2} \left( 1 + \lambda \right) - \frac{F}{2} - \frac{1}{2} \sqrt{F^2 + 2F \left( 1 + \lambda \right) + \left( 1 - \lambda \right)^2} \tag{6}$$

(II) If  $\Upsilon \leq F \leq \frac{\lambda}{1+\lambda+\sqrt{1+\lambda^2}}$ , in a symmetric mixed strategy Nash equilibrium, the tax revenue of a competing country is equal to  $\Delta \equiv \frac{F}{\lambda} \left(1 + \sqrt{1+\lambda^2}\right)$ .

(III) If F = 0, in a symmetric mixed strategy Nash equilibrium, the tax revenue of a competing country is equal to  $(1 - \lambda)$ .

It is easy and intuitive to see that a unique pure strategy Nash equilibrium exists when  $F \ge (1 + \frac{1}{\lambda})^{-1}$ . For a given  $\lambda$ , when the cost of investment in the foreign country is high, a country has to offer a very low tax rate to attract mobile agents from the competing country. Hence, the cost of setting a low tax rate to attract investment from the competing country is very high. Competing countries are segmented due to low capital mobility which induces governments set the tax rate equal to 1. Similarly we can interpret the condition for the existence of pure strategy Nash

equilibrium in terms of  $\lambda$ . For a given F, a unique pure strategy Nash equilibrium exists when  $\lambda \leq F/(1-F)$ . When  $\lambda$  is small the gain from undercutting is small since governments can only attract a mobile capital base from the competing country while the cost of offering low tax is high (due to larger  $1 - \lambda$ ). Here, it is important to emphasize the role imperfect capital mobility plays in equilibrium. When F = 0, starting from any symmetric tax rates in two competing countries, the cost of reducing tax rate to attract mobile agents from the competing country is close to zero even under the non-preferential regime.





Figure (1) displays the support of the mixed strategy Nash equilibrium when  $\lambda/\left(1 + \lambda + \sqrt{1 + \lambda^2}\right)$  $< F < \left(1 + \frac{1}{\lambda}\right)^{-1}$ , in which competing countries randomize over the intervals  $\left(1 - \Psi - F, 1 - F\right)$ and  $\left(1 - \Psi, 1\right)$ . The infimum and supremum of the support are  $1 - \Psi - F$  and 1 respectively. Noting that<sup>19</sup>  $0 < \Psi < F$ , in the mixed strategy Nash equilibrium no country sets a tax rate  $\tau$  such that  $\tau \in (1 - F, 1 - \Psi)$ . This is possible because the probability distribution of tax rates competing countries use in equilibrium is *continuous* everywhere on the support with probability mass of nonzero measure at 1. If a country sets a high tax rate then the competing country undercuts by the margin of F to attract mobile agents. When F is large enough  $\left(\lambda/\left(1 + \lambda + \sqrt{1 + \lambda^2}\right) < F\right)$ , in response to a low tax rate of the competing country, a country reduces its tax rate just enough to be able to retain its domestic mobile capital base. If a country sets tax rate equal to  $1 - \Psi$ , it is no longer revenue maximizing for the competing country to undercut. Hence,  $1 - \Psi - F$  is the infimum of the support of the mixed strategy Nash equilibrium.

If a country sets the tax rate in the intervals  $(1 - \Psi, 1)$  and  $(1 - \Psi - F, 1 - F)$ , its tax revenue depends on the tax distributions of the competing country over the intervals  $(1 - \Psi - F, 1 - F)$ and  $(1 - \Psi, 1)$ . Thus, cumulative distribution function G(.) may change its functional form over

 $<sup>\</sup>overline{\int_{1}^{19} \left(1 + \frac{1}{\lambda} + \sqrt{1 + \frac{1}{\lambda^2}}\right)^{-1}} < F < \left(1 + \frac{1}{\lambda}\right)^{-1} \Rightarrow 0 < \Psi < F.$  It is easy to check that  $F < \left(1 + \frac{1}{\lambda}\right)^{-1} \Rightarrow \Psi > 0.$ Further  $\Psi < F$  if f(F) < 0 where  $f(F) = 4\lambda - 8F(1+\lambda) + 8F^2$ . Now f''(F) = 8 > 0 and f'(F) < 0 if  $F < \frac{1+\lambda}{2}$ , which implies f'(F) < 0 when  $F < \left(1 + \frac{1}{\lambda}\right)^{-1}$ . Thus maximum of f(F) occurs at  $\left(1 + \frac{1}{\lambda} + \sqrt{1 + \frac{1}{\lambda^2}}\right)^{-1}$ . At  $\left(1 + \frac{1}{\lambda} + \sqrt{1 + \frac{1}{\lambda^2}}\right)^{-1} = F$  we have f(F) = 0. Noting that f(F) is continuous, inequality is obvious.

the two intervals. Let  $G^1(.)$  and  $G^2(.)$  be the probability distributions of tax rates a competing country use in the equilibrium over the intervals  $(1 - \Psi - F, 1 - F)$  and  $(1 - \Psi, 1)$ , respectively. In equilibrium, the expected tax revenue of a country for a tax rate  $\tau$  over the support can be written as

$$E[R(\tau)] = \begin{cases} (1-\lambda)\tau + \lambda\tau + \lambda \left[1 - G^2(\tau+F)\right], & for \ \tau \in (1-\Psi-F, \ 1-F) \\ (1-\lambda)\tau + \lambda \left[1 - G^1(\tau-F)\right], & for \ \tau \in (1-\Psi, \ 1). \end{cases}$$
(7)

If a country sets the tax rate equal to  $1 - \Psi$ , it retains its domestic mobile capital base with probability 1 and does not attract mobile capital from the competing country. Thus the expected tax revenue of competing countries in equilibrium is  $1 - \Psi$ . Because there is no probability mass anywhere on the support except at the supremum of the support, the distribution of tax over the support is continuous. Thus we have  $G^1(1 - \Psi) = G^2(1 - F)$ . Now we can find  $\Psi$ , by comparing the tax revenues of a country for the tax rates 1 and  $1 - \Psi - F$ , and using the fact that in the mixed strategy Nash equilibrium, competing countries earn equal tax revenues everywhere on the support. Because  $G^1(1 - \Psi)$  and  $G^2(1 - F)$  cancel out, we find  $\Psi$  from the above relations. Once we have  $E[R(\tau)] \equiv 1 - \Psi$ , we find the tax distributions in equilibrium using (7).



Figure (2) displays the support of the mixed strategy Nash equilibrium when  $\Upsilon \leq F \leq \lambda/\left(1+\lambda+\sqrt{1+\lambda^2}\right)$ . In equilibrium, competing countries randomize over the interval  $(\Delta - F, \Delta + F)$ . The support of the mixed strategy Nash equilibrium is convex and the supremum of the support is strictly less then 1 if  $F < \left(1 + \frac{1}{\lambda} + \sqrt{1 + \frac{1}{\lambda^2}}\right)^{-1}$ . There is no probability mass anywhere on the support. The intuition behind the equilibrium is similar to the previous case. If a country sets the tax rate  $\Delta$ , then it receives taxes from its domestic mobile and immobile capital bases with probability one while it does not attract mobile agents from the competing country. Thus in equilibrium the competing countries earn tax revenue equal to  $\Delta$ . The expected tax revenue of a competing country can be described similar to (7), when the distributions of taxes over the intervals  $(\Delta - F, \Delta)$  and  $(\Delta, \Delta + F)$  are respectively denoted as  $G^1(.)$  and  $G^2(.)$ . We find  $\Delta$  by comparing

the tax revenue of a country for the tax rates  $\Delta - F$  and  $\Delta + F$ . Note that when a country sets  $\Delta - F$ , it attracts mobile capital from the competing country with probability  $G^2(\Delta)$ . Similarly, when a country sets  $\Delta + F$ , it looses its domestic mobile capital to the competing country with probability  $G^1(\nabla)$ . We can find  $\nabla$  from the above relations using the fact that  $G^1(\nabla) = G^2(\Delta)$ . Once we know  $\triangle$ , we can find the tax distributions by using the method similar to the previous case.

Irrespective of the tax rates set by the competing country, a country can set the tax rate equal to 1 and earn  $(1 - \lambda)$  from its immobile capital base and forgo taxes from the mobile capital base completely. Hence, in equilibrium, the expected tax revenue of competing countries should be at least  $(1 - \lambda)$ . But  $F < \Upsilon$  implies  $\Delta \equiv F\left(\frac{1}{\lambda} + \sqrt{1 + \frac{1}{\lambda^2}}\right) < (1 - \lambda)$ , where  $\Upsilon$  is given by (4). When  $F < \Upsilon$ , mutual undercutting by competing countries result in a very low tax rate. Nash equilibrium in this case is not clear and we do not show existence of a mixed strategy Nash equilibrium. Assumption (1) provides a sufficient condition for the existence of a mixed strategy Nash equilibrium. It insure that the tax revenue  $\Delta \equiv F\left(\frac{1}{\lambda} + \sqrt{1 + \frac{1}{\lambda^2}}\right)$  in the mixed strategy Nash equilibrium is at least as large as  $(1 - \lambda)$ . The assumption is also displayed in figure (3). When  $\lambda$  is small, a pure strategy Nash equilibrium exists for a considerable small F, pointing out  $\Upsilon(\lambda)$  being small. To the contrary when  $\lambda$  is large, the certain tax revenue of  $(1 - \lambda)$  from the immobile capital base binds only for a very small F.



The symmetric tax game we analyze under the non-preferential regime is not reciprocally upper semi-continuous<sup>20</sup> (one of the conditions for existence of mixed strategy Nash equilibrium described in Corollary 5.2 in Reny, 1999). Note that the sum of the payoffs of two competing countries described by (3) is not upper semi-continuous; one of the conditions for existence of mixed strategy Nash equilibrium described in Dasgupta and Maskin (1986). Upper semi-continuity of the sum of the payoffs is a stronger requirement than that of reciprocal upper semi-continuity<sup>21</sup>.

<sup>&</sup>lt;sup>20</sup>Consider the strategy pair (F, 0), where country 1 sets the tax rate equal to F and country 2 sets tax rate equal to 0. Country 1 receives the tax revenue equal to  $R_{np}^1(F, 0) \equiv (1 - \lambda)F$ , and country 2 receives the tax revenue equal to  $R_{np}^2(F, 0) \equiv (1 - \lambda)F$ , and country 2 receives the tax revenue equal to  $R_{np}^2(F, 0) \equiv 0$ . Now for the strategy pair  $(F - \epsilon, 0)$ , where  $0 < \epsilon < F$ , we have  $R_{np}^1(F - \epsilon, 0) = F - \epsilon$  and  $R_{np}^2(F - \epsilon, 0) = 0$ .  $F - \epsilon > (1 - \lambda)F$  for  $1 > \lambda > 0$  and  $\epsilon < \lambda F$ . Thus we have  $R_{np}^1(F - \epsilon, 0) > R_{np}^1(F, 0)$  and  $R_{np}^2(F - \epsilon, 0) = R_{np}^2(F, 0)$ . Contradiction. <sup>21</sup>Consider a compact game  $G = (X_i, u_i)_{i=1}^2$  where X is a topological space and  $x \in X$ .  $u_i : X \to R^1$  for i = 1, 2.



Figure 4

When F = 0, the support of the mixed strategy Nash equilibrium is convex and there is no probability mass anywhere on the support. Figure (4) displays the support of the mixed strategy Nash equilibrium where 1 is the supremum and  $(1 - \lambda) / (1 + \lambda)$  is the infimum of the support. If a country sets the tax rate equal to  $(1 - \lambda) / (1 + \lambda)$ , it retains its domestic capital base and also attracts mobile agents from the competing country with probability 1. Thus, in equilibrium, the tax revenue of a competing country is equal to  $(1 - \lambda)$ . In this case, starting from any symmetric tax rates set by the competing countries, the cost of offering a low tax rate to attract mobile agents from the competing country is close to zero explaining why in equilibrium, competing countries earn the tax revenue that equals the amount they can earn simply by setting the tax rate equal to 1 on the immobile capital while forgoing taxes from the mobile capital base completely. See Varian (1980) for a complete characterization of a mixed strategy Nash equilibrium of similar nature. Wilson (2005) also obtains a similar result in his analysis when one of the capital bases is perfectly mobile. In our model the tax revenue of a competing country in an equilibrium is at least as large as  $(1-\lambda)$ . The result differs due to the fact that a country has to undercut the tax rate of the competing country by a positive margin of F to attract mobile agents. Thus, imperfect capital mobility not only reduces gains from undercutting, it also reduces the tax revenue from the immobile capital base (cost of undercutting). These two effects together reduce competition, thereby increasing tax revenues.

**Proposition (2):** Under a non-preferential regime tax revenues of competing countries decrease monotonically as F decreases or  $\lambda$  increases.

Suppose  $u \equiv \sum_{i=1}^{i=2} u_i$  is upper semi-continuous on X, and let us assume that, there exist  $(\bar{x}, \bar{u}) \in cl(gr(u))$  such that  $u_i(x) \leq \bar{u}_i \quad \forall i$ , and  $u(\bar{x}) \neq \bar{u}$ . Then,  $u_1(\bar{x}) + u_2(\bar{x}) < \bar{u}_1 + \bar{u}_2$ . Consider a sequence  $(x_1^n, x_2^n) \subseteq X$ , such that  $(x_1^n, x_2^n) \to (\bar{x}_1, \bar{x}_2)$ . Since  $(\bar{x}, \bar{u}) \in cl(gr(u))$  lim  $\sup_{x^n \to \bar{x}} u_i(x_1^n, x_2^n) = \bar{u}_i$ . Then  $\limsup_{x^n} u(x^n) = \bar{u}_1 + \bar{u}_2$ . On the other hand,  $\limsup_{x^n \to \bar{x}} u(x^n) \leq u_1(\bar{x}) + u_2(\bar{x})$ , because u is upper semi-continuous on X. Hence,  $\bar{u}_1 + \bar{u}_2 \leq u_1(\bar{x}) + u_2(\bar{x})$ . Contradiction. Thus, if u is upper semi-continuous, then it must also be reciprocally upper semi-continuous.

The result of proposition (2) is not surprising. If  $F < (1 + \frac{1}{\lambda})^{-1}$ , then a pure strategy Nash equilibrium does not exist and the competing countries lower the tax rate to attract mobile capital from the competing country. As F decreases it becomes less costly for mobile agents to relocate capital in the country with lower tax rate. Similarly, larger  $\lambda$  provides more incentive for competing countries to offer a lower tax rate to attract mobile capital from the competing country. Also, the loss of tax revenue from the immobile capital base due to lower tax is small for a large  $\lambda$ .

#### 4 Preferential Regime

Under the preferential regime, competing countries set different tax rates for mobile and immobile capital bases. Irrespective of F, competing countries set the maximum possible tax rate (equal to 1) on the immobile capital base therefore in this section, unless until it is specifically mentioned, a tax rate refers to the tax rate on the mobile capital base. Since the fraction  $(1 - \lambda)$  of total capital is immobile, both countries earn tax revenues equal to  $(1 - \lambda)$  from the immobile capital base. For a given F, competition for the mobile capital base is more intense. The tax revenue a country earns from the mobile capital base depends on F, and the tax rate of the competing country on the mobile capital base. Let  $\tau^i$  and  $\tau^j$  be the tax rates on the mobile capital base set by country i and j respectively. Then the gross tax revenue of country i can be described as

$$R_{p}^{i}\left(\tau^{i}, \tau^{j}\right) = (1-\lambda) + \begin{cases} \lambda\tau^{i}, & \text{if } \left|\tau^{i}-\tau^{j}\right| < F\\ 2\lambda\tau^{i}, & \text{if } \tau^{i} \le \tau^{j}-F\\ 0, & \text{if } \tau^{i} \ge \tau^{j}+F, \end{cases}$$
(8)

where  $i, j = \{1, 2\}$  and  $i \neq j$ . If  $|\tau^i - \tau^j| < F$ , then mobile agents invest in the country of their residence. When  $\tau^i \leq \tau^j - F$ , mobile agents in both countries invest in country i. Since fraction  $\lambda$ of the total capital is mobile, country i earns tax revenues equal to  $2\lambda\tau^i$  from mobile agents and country j does not receive taxes from mobile agents. The outcome is reversed when  $\tau^i \geq \tau^j + F$ . In this case mobile agents in both competing countries invest in country j. Note that the payoff function described by (8) is not quasi-concave<sup>22</sup> for 0 < F < 1.

Tax competition for the mobile capital base is similar to a symmetric Bertrand price competition in which a competitor has to undercut the tax rate/price set by a rival entity by a positive discrete margin F. If the cost of undercutting the rival country is sufficiently small then a pure strategy Nash equilibrium does not exist. A country desire a low tax rate when the tax rate of the competing country is high and also desire a high tax rate when the tax rate of the competing country is low. We find a mixed strategy Nash equilibrium of this tax competition for the mobile capital base by using a method similar to proposition (1). Under the preferential regime we find a Nash equilibrium

 $<sup>\</sup>overline{1 - F + \epsilon \in (1, 1 - F) \quad \text{s.t. } 0 < \epsilon < F. \quad R_p^i(1, 1) = 1 \text{ and } R_p^i(1 - F, 1) = (1 - \lambda) + (1 - F) 2\lambda > 0. \text{ Now consider,}} \\ 1 - F + \epsilon \in (1, 1 - F) \quad \text{s.t. } 0 < \epsilon < F. \quad R_p^i(1 - F + \epsilon, 1) = 1 - F + \epsilon. \text{ Clearly, } 1 - F + \epsilon < 1. \text{ Now suppose}} \\ (1 - \lambda) + (1 - F) 2\lambda \leq 1. \text{ But } 1 - F + \epsilon < (1 - \lambda) + (1 - F) 2\lambda \text{ for } \epsilon < F + \lambda - 2\lambda F. \text{ Now } F + \lambda - 2\lambda F \equiv F(1 - \lambda) + \lambda(1 - F) > 0 \text{ for } \lambda, \quad F \in (0, 1). \text{ Hence, we show that } R_p^i(\tau^i, \tau^j) \text{ is not quasi-concave. By symmetry} \\ \text{the same is true for } R_p^j(\tau^i, \tau^j).$ 

for all values<sup>23</sup> of  $\lambda$  and F.

**Proposition (3):** In equilibrium, a competing country sets a tax rate equal to 1 on the immobile capital base from which the tax revenue is equal to 1. The tax rate on the mobile capital base depends on F. If  $F \ge 1/2$ , a unique pure strategy Nash equilibrium exists. In equilibrium, a competing country sets the tax rates equal to 1, in which case the tax revenue is equal to  $\lambda$ . If 0 < F < 1/2, a pure strategy Nash equilibrium does not exists, however a mixed strategy Nash equilibrium exists.

(I) If  $1/(2 + \sqrt{2}) < F < 1/2$ , in a symmetric mixed strategy Nash equilibrium, the expected tax revenue of a competing country is equal to  $\lambda(1 - \Phi)$ , Where

$$\Phi = 1 - \frac{F}{2} - \frac{1}{2}\sqrt{F^2 + 4F}.$$
(9)

(II) If  $0 < F \leq 1/(2 + \sqrt{2})$ , in a symmetric mixed strategy Nash equilibrium the expected tax revenue of a competing country is equal to  $\lambda F(1 + \sqrt{2})$ .

(III) If F = 0, a unique pure strategy Nash equilibrium exists where a competing country sets tax rate equal to 0.

Governments compete head-to-head to attract mobile agents from the competing country. In equilibrium, tax rates on the mobile capital base depends only on the maximum possible tax rate (which is equal to 1) and F. It is easy and intuitive to see that a pure strategy Nash equilibrium exists if  $F \ge 1/2$ . The cost of moving capital is so high that it is beneficial for the competing countries to concentrate on taxing domestic agents only. Because there is no threat of mobile agents moving abroad, the competing countries are completely segmented. Hence, governments set the highest possible tax rate on the mobile capital base as well.

When F < 1/2, a pure strategy Nash equilibrium does not exist. The reason for the nonexistence of a pure strategy Nash equilibrium is clear from the discussion in section (3). Note that if we substitute  $\lambda = 1$  in proposition (1) we get the distributions of tax rates competing countries use in the equilibrium under the preferential regime. We find the mixed strategy Nash equilibrium using a method similar to that for proposition (1).

When  $1/(2 + \sqrt{2}) < F < 1/2$ , in a symmetric mixed strategy Nash equilibrium a competing country randomizes over the intervals  $(1 - \Phi - F, 1 - F)$  and  $(1 - \Phi, 1)$ . Figure (1) displays the support of the mixed strategy Nash equilibrium where we replace  $\Psi$  with  $\Phi$  to identify the intervals in this case, where  $\Phi$  is given by (9). Note that  $^{24} \Phi < F$ , implying that in the equilibrium no country sets a tax rate  $\tau$ , such that  $1 - F < \tau < 1 - \Phi$ . This is possible because there is a probability mass of nonzero measure at 1. If a country sets tax rate equal to  $1 - \Phi$ , it retains the domestic mobile capital base with probability 1 and it does not attract mobile agents from the competing country because  $\Phi < F$ . Thus in equilibrium, a competing country earns the tax revenue equal to  $1 - \Phi$ .

<sup>&</sup>lt;sup>23</sup>Note that under preferential regime countries sets tax rates for mobile and immobile capital independent of each other. Thus  $\lambda$  does not play any role in competition for mobile capital. Putting  $\lambda = 1$  in (2) we get  $F \ge 0$ , which is trivially satisfied.

 $<sup>^{24}\</sup>Phi < F \Leftrightarrow f(F) \equiv 8F^2 + 4 - 10F > 0$ . Now f(F) can be written as 8F(F - 1/4) + 8(1/2 - F). Hence f(F) > 0 as  $1/(2 + \sqrt{2}) < F < 1/2$ .

If  $F \leq 1/(2 + \sqrt{2})$ , in a symmetric mixed strategy Nash equilibrium, a competing country randomizes over the intervals  $[\sqrt{2}F, F(2 + \sqrt{2})]$ . The support of the mixed strategy Nash equilibrium is convex with no probability mass anywhere on the support. Figure (2) displays the support of the mixed strategy Nash equilibrium. We can identify the infimum and the supremum of the support in figure (2) by substituting  $\lambda = 1$ . Note that if a country sets the tax rate equal to  $(1 + \sqrt{2})F$ it receives taxes from its domestic mobile agents with probability 1 and it does not attract mobile agents from competing country. Hence, the tax revenue of a competing country from the mobile capital base in the equilibrium is equal to  $\lambda (1 + \sqrt{2})F$ .

The mixed strategy Nash equilibrium described in proposition (3) has desirable properties which explain existing variations in tax rates on mobile capital bases even among countries which have not yet committed to a non-preferential taxation agreement. When the cost of capital relocation is sufficiently high competing countries set their tax rate equal to 1 with strictly positive probability. Also as long as the cost of capital relocation is strictly positive, competing countries earn strictly positive tax revenues from the mobile capital base as well. Even though competition among independent countries have reduced tax rates on mobile capital bases a considerable extent still even today most countries impose strictly positive tax rates on highly mobile capital. Perfect capital mobility emerges as the limiting case of our current analysis when the cost of capital relocation is zero. Perfect capital mobility between countries leads to a Bertrand type outcome where the mobile capital base is not taxed.

**Proposition (4):** Under the preferential regime tax revenues of competing countries decrease monotonically as F decreases or  $\lambda$  increases.

Proposition (4) is similar to proposition (2). If F < 1/2, a pure strategy Nash equilibrium does not exist. Competing countries reduce tax rate on the mobile capital base to attract mobile agents from the competing country. As F decreases it becomes easier for mobile agents to invest in a country with the lower tax rate. Increases in  $\lambda$  affect tax revenues of the competing countries directly. Countries compete for a bigger fraction of the total capital. Because total capital is held constant, this results in lower tax revenues from the immobile capital base.

## 5 Comparison

In this section we compare the tax revenues of competing countries under the preferential and the non-preferential regime. First we formally compare tax revenues from the mobile capital base under the two regimes.

**Proposition (5):** The tax revenue from the mobile capital base under the non-preferential regime is as large as under the preferential regime. When  $F \ge 1/2$ , tax revenues under the two regimes are equal, while the tax revenue is strictly higher under the non-preferential regime when F < 1/2.

We can see that when the restriction on capital flow is not large, competing countries earn more from the mobile capital base. This confirms the common belief in the literature that a non-preferential regime lowers the competition for the mobile capital base therefore increase tax revenues. But as Keen (2001) argues a non-preferential regime spreads competition for a mobile capital base to a relatively less mobile capital base, which reduces tax revenue. This effect is evident in proposition (6), where we formally compare tax revenues of competing countries from the immobile capital base under the two regimes.

**Proposition (6):** The tax revenue from the immobile capital base under the preferential regime is as large as under the non-preferential regime. When  $F \ge \left(1+\frac{1}{\lambda}\right)^{-1}$ , tax revenues under the two regimes are equal, while the tax revenue is strictly higher under the preferential regime when  $F < \left(1+\frac{1}{\lambda}\right)^{-1}$ .

Proposition 5 – 6 emphasize the trade off from adopting a non-preferential vs a preferential regime. When the cost of capital relocation is very large, countries are segmented and there is no competition to attract mobile agents from a foreign country implying that a preferential or a non-preferential regime does not make a difference as long as the tax revenue is concern. But when capital is mobile enough competing countries earn higher tax revenues from the mobile capital base under the non-preferential regime. Head-to-head competition for the mobile capital lowers tax revenues from the mobile capital base under the preferential regime when compared to the non-preferential regime. This is easy to see when  $1/2 < F \leq (1 + \frac{1}{\lambda})^{-1}$ . Under the non-preferential

regime a pure strategy Nash equilibrium exists, and competing countries earn tax revenues equal to  $\lambda$  from the mobile capital base. When governments follow a preferential regime, a pure strategy Nash equilibrium does not exist. In the mixed strategy Nash equilibrium competing countries earn  $\lambda (1 - \Phi)$ , which is strictly less then  $\lambda$ . For any given  $\lambda$ , this is true for all values of F. This is the basis of arguments against a preferential regime. But under a preferential regime countries are able to earn higher tax revenues from the immobile capital base when  $\Upsilon \leq F < (1 + \frac{1}{\lambda})^{-1}$ . In this case under the non-preferential regime pure strategy Nash equilibrium does not exist and competing governments set tax rates below 1 with strictly positive probability. In contrast under the preferential regime the tax rate on the immobile capital is fixed at 1. Whether the non-preferential or the preferential regime generate higher tax revenues depends on which of the two effects discussed above dominates.

**Proposition (7):** When F = 0 or  $F \ge 1/2$ , the tax revenues under the two regimes are equal. A preferential regime generates higher tax revenues when  $F^1 \le F \le F^2$ , and a non-preferential regime generates higher tax revenues when  $1/2 > F > F^2$ , where

$$F^{1} = (1-\lambda) \left[ \frac{1}{\lambda} + \sqrt{1+\frac{1}{\lambda^{2}}} \right]^{-1}$$
and (10)

$$F^{2} = (1-\lambda) \left[ \left( \frac{1}{\lambda} + \sqrt{1+\frac{1}{\lambda^{2}}} \right) - \lambda \left( 1 + \sqrt{2} \right) \right]^{-1}.$$
 (11)

The first part of the proposition (7) confirms the proposition (1) in Wilson (2005). If a fraction of the total capital base is perfectly mobile, the other fraction of capital base is perfectly immobile and competing countries are identical; both a non-preferential and a preferential regime generates equal tax revenues. The comparison differs when the mobile capital base is imperfectly mobile. We can see that when F is sufficiently large; *i.e.*,  $1/2 > F > F^2$ , the competing countries earn higher tax revenues under the non-preferential regime. The opposite is true when  $F^1 \leq F < F^2$ , except at the point of equality,  $F = F^2 > 0$ , in which case competing countries earn equal tax revenues under the two regimes. We provide the following explanation for results in proposition (7). For any given  $\lambda$ , when F is large, the positive effect of higher tax revenues from the mobile capital base due to less competition under the non-preferential regime dominates. The opposite is true when F is relatively small. The competing countries have to set low tax rates on the mobile and the immobile capital bases due to competition for the mobile capital base. The loss of tax revenue arising from low tax rates on the immobile capital is more in comparison to gains in tax revenues from higher tax rates on the mobile capital base.

It is noteworthy that under the non-preferential regime when  $F = F^2$ , the competing countries earn tax revenues equal to  $(1 - \lambda)$  in equilibrium. Even though F is strictly positive, competition for the mobile capital base drives down the expected tax revenues of competing governments to what they can earn simply by setting tax rate equal to 1 and receive taxes only from their domestic immobile agents. On the other hand under the preferential regime the competing countries obtains strictly positive tax revenues from the mobile capital base as long as F is strictly positive.

$F^1$	$F^2$	$\lambda$
.044888	.045435	0.1
.079216	.083193	0.2
0.10274	0.11496	0.3
0.11555	0.14195	0.4
0.118 03	0.16507	0.5
0.11079	0.18503	0.6
.094567	0.20237	0.7
.070156	0.21753	0.8
.038374	0.23086	0.9
Table 1		

In table (1) we list  $F^1(\lambda)$  and  $F^2(\lambda)$  for different values of  $\lambda$ . The preferential regime generates higher tax revenue compared to the non-preferential regime under reasonable circumstances. For example if 70% of the total capital is mobile then the competing countries earn higher tax revenues under the preferential regime if .094567  $\leq F < 0.20237$  and earn higher tax revenues under the non-preferential regime if 0.5 > F > 0.20237.



Figure 5

We display results of proposition (7) again in figure (5). Function  $F^1(\lambda)$  indicates for a given  $\lambda$ , the minimum value of F which insure that we have a mixed strategy Nash equilibrium under the non-preferential regime. For a given  $\lambda$  distance between  $F^1(\lambda)$  and  $F^2(\lambda)$  marks the range of F for which the preferential regime generates a higher tax revenue compared to the non-preferential regime. Similarly for F above  $F^2(\lambda)$  but less then 1/2, the non-preferential regime generates a higher tax revenue compared to the preferential regime.  $F^2(\lambda)$  is monotonically increasing with  $\lambda$  The reason is that the preferential regime dominates the non-preferential regime when competition for mobile capital base is intense; an increase in  $\lambda$  intensifies competition under the non-preferential regime.

### 6 Conclusion

Literature on tax competition identifies three factors – home bias, endogenous size of capital base, and asymmetry, that may explain the gain from a non-preferential agreement following surprising result in Keen (2001) in which the restriction on a preferential taxation strategy reduces tax revenues of competing countries. We identify imperfect capital mobility as another source of gains from a non-preferential regime. In a symmetric game of tax competition we show that a non-preferential regime generates higher tax revenue if capital is not highly mobile. On the other hand when capital is highly mobile and mobile capital captures relatively large fraction of total capital a preferential regime generates higher tax revenues. When mobility of capital is extremely low and when mobile capital base is perfectly mobile, then a non-preferential and a preferential regime generates equal tax revenues. When mobile capital is imperfectly mobile our model gives more realistic prediction, where, in equilibrium the competing countries can obtain mobile capital even if governments set different tax rates. Also our model describes when the non-preferential (preferential) regime generates higher tax revenue compared to the preferential (non-preferential) regime in terms of parameters of the model.

We do not impose restrictions to insure the existence of a pure strategy Nash equilibrium. The mixed strategy Nash equilibrium when competing nations/firms have immobile capital base/captive segment is well understood when a nation/firm can attract investors/consumers by undercutting the tax rate/price set by a competing nation/firm by a arbitrary small margin. We extend such equilibrium to the case in which a nation/firm has to undercut the tax rate/price set by a competing nation/firm by a discrete positive margin. These results should be useful to a much larger literature in applied game theory and industrial organization.

## 7 Appendix

**Proof of Proposition (1):** The strategy pair (1, 1) is a pure strategy Nash equilibrium if no country can set a tax rate lower than 1 and do strictly better. If a country sets the tax rate 1 - F, it will earn tax revenues equal to  $1 - F + \lambda (1 - F)$ . For the strategy pair (1, 1), to be a pure strategy Nash equilibrium it must be true that  $1 \ge 1 - F + \lambda (1 - F)$ , which implies  $F(1 + \frac{1}{\lambda}) \ge 1$ . Thus the strategy pair (1, 1) is a Nash equilibrium if  $F \ge (1 + \frac{1}{\lambda})^{-1}$ . By contradiction, we show that a pure strategy Nash equilibrium does not exist if  $F < (1 + \frac{1}{\lambda})^{-1}$ . It is clear that (1, 1) is not a pure strategy Nash equilibrium if  $F < (1 + \frac{1}{\lambda})^{-1}$ . Suppose the strategy pair  $(\tau, \tau)$  is a pure strategy Nash equilibrium, where  $\tau < 1$ . But a country can set the tax rate  $\tau' = \min\{1, \tau + F - \epsilon\}$ , for very small  $\epsilon > 0$ , and do strictly better. Thus, a symmetric pure strategy Nash equilibrium does not exist the strategy pair (1, 1) is a unique pure strategy Nash equilibrium the strategy better. Thus, a symmetric pure strategy Nash equilibrium does not exist. From the discussion this is also clear that the strategy pair (1, 1) is a unique pure strategy Nash equilibrium when  $F(1 + \frac{1}{\lambda}) \ge 1$ .

Again by contradiction we show that an asymmetric pure strategy Nash equilibrium does not exist. Suppose the strategy pair  $(\tau^1, \tau^2)$  is a Nash equilibrium, where  $\tau^1$  and  $\tau^2$  are tax rates set by country 1 and country 2. Without loss of generality suppose  $\tau^1 < \tau^2$ . Then it must be true that  $\tau^1 = \tau^2 - F$ . Because, if  $\tau^1 < \tau^2 - F$  then country 1 can do better by setting the tax rate equal to  $\tau^1 = \tau^2 - F$ . But the strategy pair  $(\tau^2 - F, \tau^2)$  is not a Nash equilibrium because country 2 can do better by decreasing the tax rate slightly. Also if  $\tau^2 - F < \tau^1 < \tau^2$ , then country 1 can certainly do better by increasing its tax rate to  $\tau^2$ . This contradicts the fact that the strategy pair  $(\tau^1, \tau^2)$  is a pure strategy Nash equilibrium. Thus we prove that if  $F < (1 + \frac{1}{\lambda})^{-1}$ , a pure strategy Nash equilibrium does not exist.

**part** (I): When  $\lambda/(1 + \lambda + \sqrt{1 + \lambda^2}) < F < (1 + \frac{1}{\lambda})^{-1}$ , in mixed strategy Nash equilibrium competing countries randomize over the intervals  $(1 - \Psi - F, 1 - F)$  and  $(1 - \Psi, 1)$ . The distribution of tax rates a country use in the equilibrium is  $G^1(\tau)$  for  $\tau \in (1 - \Psi - F, 1 - F)$  and  $G^2(\tau)$  for  $\tau \in (1 - \Psi, 1)$ . There is probability mass of  $m^p$  at 1, where:

$$G^{1}(\tau) = \begin{cases} 0 & \text{for } \tau \leq 1 - \Psi - F.\\ 1 - \frac{\Omega^{N} - (1 - \lambda)(\tau + F)}{\lambda(\tau + F)} & \text{for } \tau \in (1 - \Psi - F, \ 1 - F) \end{cases}$$
(A1)

$$G^{2}(\tau) = \left\{ 1 - \frac{\Omega^{N} - (\tau - F)}{\lambda(\tau - F)} \quad \text{for } \tau \in (1 - \Psi, 1) \right.$$
(A2)

 $m^p \equiv \frac{\Omega^N - (1-F)}{\lambda(1-F)}$  and  $\Psi$  is given by (6).  $\Omega^N \equiv 1 - \Psi$  is the tax revenue of a competing country in the mixed strategy Nash equilibrium.

Because the equilibrium is symmetric we do not use a superscript to differentiate between the two countries. First of all we need to show that

$$\lim_{\epsilon \to 0} G\left(1 - F - \epsilon\right) = \lim_{\epsilon \to 0} G\left(1 - \Psi + \epsilon\right).$$
(A3)

Suppose (A3) holds if the expected tax revenue of a competing countries in the mixed strategy Nash equilibrium is equal to some arbitrary amount  $\Omega$ . Then from (A1) - (A3) we have:

$$1 - \frac{\Omega - (1 - \lambda)((1 - F) + F)}{\lambda((1 - F) + F)} = 1 - \frac{\Omega - ((1 - \Psi) - F)}{\lambda((1 - \Psi) - F)}.$$
 (A4)

Solving for  $\Omega$  from (A4) we get  $\Omega = \Omega^N$ . Thus, (A3) holds.

The tax distribution over the interval  $(1 - \Psi, 1)$  is given by (A2). Therefore,

$$m^{p} \equiv 1 - G(1) = \frac{\Omega^{N} - (1 - F)}{\lambda (1 - F)} > 0$$
 (A5)

Equation (A5) confirms that the distribution of tax rates over the support has probability mass of  $\frac{\Omega^N - (1-F)}{\lambda(1-F)}$  at 1. We prove the remaining part in two steps. In step one we show that a competing country earn an equal tax revenue everywhere on the support. Subsequently, in step two we show that a country can not deviate unilaterally and do strictly better.

Step One – The expected tax revenue of a country if it sets the tax rate equal to 1 is

$$\Omega = (1 - \lambda) 1 + \lambda [1 - G (1 - F)]$$
(A6)

$$= (1-\lambda) + \lambda \frac{\Omega^N - (1-\lambda)\left((1-F) + F\right)}{\lambda\left((1-F) + F\right)} = \Omega^N.$$
(A7)

Similarly for the tax rate equal to (1 - F), the expected tax revenue of a competing country is equal to

$$\Omega = 1 - F + \lambda m^p \left( 1 - F \right). \tag{A8}$$

In this case the country retains its domestic mobile capital base with probability 1 and also attracts mobile agents of the competing country with probability  $m^p$ . Substituting for  $m^p$  in (A8) we get  $\Omega = \Omega^N$ . Similarly if a country sets a tax rate  $\tau$  s.t.  $\tau \in (1 - \Psi - F, 1 - F)$ , then its expected tax revenue is

$$\Omega = \tau + \lambda \tau \left[ 1 - G \left( \tau + F \right) \right]. \tag{A9}$$

Note that  $\tau \in (1 - \Psi - F, 1 - F) \Rightarrow \tau + F \in (1 - \Psi, 1)$ . Using (A3) and (A9) we get  $\Omega = \Omega^N$ . Similarly if a country sets a tax rate  $\tau$  such that  $\tau \in (1 - \Psi, 1)$ , its expected tax revenue is

$$\Omega = (1 - \lambda)\tau + \lambda\tau \left[1 - G(\tau - F)\right].$$
(A10)

where  $G(\tau - F)$  is given by (A2) since  $\tau \in (1 - \Psi, 1) \Rightarrow \tau - F \in (1 - \Psi - F, 1 - F)$ . Thus using (A10) and (A2) we get  $\Omega = \Omega^N$ . This proves that the expected tax revenue of a competing country is equal everywhere on the support.

Step Two - Now we show that a country can not do better by a unilateral deviation. The supremum of the support is 1. Therefore, we only need to check that a country can not do better if it sets a tax rate lower then the minimum of the support. Suppose country a deviates and set a tax rate  $\tau$  such that  $1 - \Psi - F < \tau \leq 1 - 2F$ . The expected tax revenue of the country is  $\Omega \equiv \tau + \lambda \tau [1 - G(\tau + F)]$ . Since in the equilibrium no country sets a tax rate in the range  $(1 - F, 1 - \Psi)$ , deviating country does strictly worse. This is because the country can raise the tax rate to the infimum of the support and still collect taxes from the domestic mobile agents with probability 1 and from mobile agents of the competing country with probability  $G^2 (1 - \Psi)$ .

Now suppose a country deviates and sets a tax rate  $\tau$  such that  $1 - \Psi - 2F < \tau < 1 - 2F$ . The deviating country is able to attract mobile agents of the competing country with probability  $[1 - G(\tau + F)]$ , where  $G(\tau + F)$  is given by (A1). Note that,  $\tau \in (1 - \Psi - 2F, 1 - 2F) \Rightarrow \tau + F \in (1 - \Psi - F, 1 - F)$ . Using (A1), the expected tax revenue of the deviating country is equal to:

$$\Omega = \tau + \lambda \tau \frac{\Omega^N - (1 - \lambda) (\tau + F)}{\lambda (\tau + F)},$$
  

$$\Rightarrow \frac{\partial \Omega}{\partial \tau} = \lambda + \Omega^N \left[ \frac{F}{(\tau + F)^2} \right] > 0.$$
(A11)

From (A11) it is clear that for  $\tau \in (1 - \Psi - 2F, 1 - 2F)$ , the deviating country maximizes its expected tax revenue at 1 - 2F. But at 1 - 2F, a country earns less compared to the expected tax revenue in the mixed strategy Nash equilibrium. Hence the deviating country can not do better by setting a tax rate in the range  $(1 - \Psi - 2F, 1 - 2F)$ .

Now we only need to show that a country can not do better if it sets the tax rate equal to  $1-\Psi - 2F$ . In this case the deviating country undercuts the competing country by the margin of F with probability 1. The tax revenue of the deviating country in this case is equal to  $(1 + \lambda) (1 - \Psi - 2F)$ . We need to show that

$$(1+\lambda)(1-\Psi-2F) \leq 1-\Psi$$
  
$$\Rightarrow 1-\Psi \leq 2F\left(1+\frac{1}{\lambda}\right).$$
(A12)

Note that  $1 < \frac{F}{\lambda} \left( 1 + \lambda + \sqrt{1 + \lambda^2} \right) \Rightarrow 1 - F < F \left( \frac{1}{\lambda} + \sqrt{1 + \frac{1}{\lambda^2}} \right)$ . Also we know that  $\Psi < F$ . Therefore, a sufficient condition for (A12) to hold is:

$$2F\left(1+\frac{1}{\lambda}\right) \ge F\frac{1}{\lambda}\left(1+\lambda\sqrt{1+\lambda^2}\right) \Rightarrow \left(1+\frac{1}{\lambda}\right) \ge \sqrt{1+\frac{1}{\lambda^2}},$$

which is true since  $0 < \lambda < 1$ . Thus we prove that a competing can not do better by a unilateral deviation. By symmetry the same is true for the other country as well. This completes the proof of the part (I) of the proposition (1).

**part (II):** When  $\Upsilon \leq F \leq \lambda / (1 + \lambda + \sqrt{1 + \lambda^2})$ , the distribution of tax rates a competing country uses in the mixed strategy Nash equilibrium is:

$$G^{1}(\tau) = \begin{cases} 0 & \text{for } \tau \leq \underline{\tau} \\ 1 - \frac{\Delta - (1 - \lambda)(\tau + F)}{\lambda(\tau + F)} & \text{for } \tau \in (\underline{\tau}, \ \underline{\tau} + F) \end{cases}$$
(A13)

$$G^{2}(\tau) = \begin{cases} 1 - \frac{\Delta - (\tau - F)}{\lambda(\tau - F)} & \text{for } \tau \in (\underline{\tau} + F, \ \underline{\tau} + 2F) \\ 1 & \text{for } \tau \ge \underline{\tau} + 2F \end{cases}$$
(A14)

where  $\Delta \equiv \frac{F}{\lambda} \left( 1 + \sqrt{1 + \lambda^2} \right)$  is the expected tax revenue of a competing country in the equilibrium.

First of all we show that the distribution functions described by (A13) and (A14) is continuous over the support; i.e.,

$$\lim_{\epsilon \to 0} G\left(\underline{\tau} + F - \epsilon\right) = \lim_{\epsilon \to 0} G\left(\underline{\tau} + F + \epsilon\right).$$
(A15)

One can calculate  $\lim_{\epsilon \to 0} G(\underline{\tau} + F - \epsilon)$  and  $\lim_{\epsilon \to 0} G(\underline{\tau} + F + \epsilon)$  from (A13) and (A14), respectively. Let (A15) holds if  $\Delta$  in (A13) and (A14) are replaced with some arbitrary amount  $\Omega$ . Solving (A15) for  $\Omega$ , we get

$$\frac{\Omega - (1 - \lambda)\left(\underline{\tau} + F + F\right)}{\lambda\left(\underline{\tau} + F + F\right)} = \frac{\Omega - (\underline{\tau} + F - F)}{\lambda\left(\underline{\tau} + F - F\right)} \Rightarrow \Omega = F\left(\frac{1}{\lambda} + \sqrt{1 + \frac{1}{\lambda^2}}\right),$$

which is equal to the expected tax revenue of a competing country in the mixed strategy Nash equilibrium. This proves that the distribution of tax rates over the support is continuous.

The remaining part of the proof we state in two steps. In step one we show that the expected tax revenue of a competing country is equal everywhere on the support. Subsequently, in step two we show that a country can not do strictly better by a unilateral deviation given that the other country sticks with the proposed equilibrium strategy.

STEP ONE - If a competing country sets a tax rate  $\tau$  such that  $\tau \in (\underline{\tau}, \underline{\tau} + F)$ , then its expected tax revenue is  $\tau + \lambda \tau [1 - G(\tau + F)]$ . Since  $\tau \in (\underline{\tau}, \underline{\tau} + F)$  implies  $\tau + F \in (\underline{\tau} + F, \underline{\tau} + 2F)$ . Using

(A14), the expected tax revenue of a competing country is

$$\Omega \equiv \tau + \lambda \tau \left[1 - G \left(\tau + F\right)\right]$$
  
=  $\tau + \lambda \tau \left[\frac{\Delta - \tau}{\lambda \tau}\right] = \Delta$  (A16)

If a country sets a tax rate  $\tau$  such that  $\tau \in (\underline{\tau} + F, \underline{\tau} + 2F)$ , its expected tax revenue is  $(1 - \lambda)\tau + \lambda\tau [1 - G(\tau - F)]$ . Since  $\tau \in (\underline{\tau} + F, \underline{\tau} + 2F)$  implies  $\tau - F \in (\underline{\tau}, \underline{\tau} + F)$ . Using (A13), the expected tax revenue of the competing country is:

$$\Omega \equiv (1 - \lambda)\tau + \lambda\tau \left[\frac{\Delta - \tau (1 - \lambda)}{\lambda\tau}\right] = \Delta.$$
(A17)

From (A16) and (A17), it is clear that the expected tax revenue of a competing country is equal everywhere on the support.

STEP Two - Now we show that a country can not do strictly better by an unilateral deviation. Suppose a country deviates and sets a tax rate  $\tau$  such that  $\tau \in (\underline{\tau} + 2F, \underline{\tau} + 3F)$ . Note that  $\tau \in (\underline{\tau} + 2F, \underline{\tau} + 3F) \Rightarrow (\tau - F) \in (\underline{\tau} + F, \underline{\tau} + 2F)$ . The expected tax revenue of the competing country is  $\Omega \equiv (1 - \lambda)\tau + \lambda\tau [1 - G(\tau - F)]$ . Thus, using (A14) we have

$$\Omega = \frac{\Delta - \lambda}{\tau - 2F - \lambda} \Rightarrow \frac{\partial \Omega}{\partial \tau} = -\frac{\Delta - \lambda}{\left(\tau - 2F - \lambda\right)^2} < 0.$$
(A18)

From (A18), it is clear that for  $\tau \in (\underline{\tau} + 2F, \underline{\tau} + 3F)$ , the deviating country maximizes its expected tax revenue at the infimum of the set. If the deviating country sets a tax rate higher than  $\underline{\tau} + 3F$ , it looses its mobile capital to the other country with probability 1. Thus the maximum tax revenue the deviating country can earn for a tax rate higher than  $\underline{\tau} + 3F$  is  $(1 - \lambda)$ , which is lower then the expected tax revenue of the country in mixed strategy Nash equilibrium. Noting that the tax distribution over the support a competing country use in the equilibrium is continuous, it is clear that the deviating country can not do better by setting a tax rate higher than the supremum of the support.

Now we show that the deviating country can not do better by setting a tax rate lower than  $\underline{\tau}$ . If the country sets a tax rate in the range  $(\underline{\tau} - F, \underline{\tau})$ , its expected tax revenue is  $\tau + \lambda \tau [1 - G(\tau + F)]$ . Since  $(\tau + F) \in (\underline{\tau}, \underline{\tau} + F)$  the expected tax revenue of the deviating country can be found using (A13), which is equal to

$$\Omega = \tau + \lambda \tau \left[ \frac{\Delta - (1 - \lambda) (\tau + 2F)}{\lambda (\tau + 2F)} \right] = \lambda \tau + \Delta \frac{\tau}{\tau + 2F},$$
  

$$\Rightarrow \frac{\partial \Omega}{\partial \tau} = \lambda + 2 \frac{\Delta F}{(\tau + 2F)^2} > 0.$$
(A19)

From (A19), it is clear that the deviating country can not do better by setting a tax rate lower than the infimum of the support. This completes the proof of the part (II) of the proposition (1).

**part (III):** If F = 0, then the distribution of tax rates a competing country use in the mixed strategy Nash equilibrium is:

$$G(\tau) = \begin{cases} 1 - \frac{(1-\lambda)(1-\tau)}{\lambda\tau} & \text{for } \tau \in \left(\left(\frac{1-\lambda}{1+\lambda}\right), 1\right) \\ 0 & \text{for } \tau \le \left(\frac{1-\lambda}{1+\lambda}\right). \end{cases}$$
(A20)

In equilibrium, the expected tax revenue of a competing country is equal to  $(1 - \lambda)$ .

First of all we show the expected tax revenue of a competing country is equal everywhere on the support. If a competing country sets a tax rate  $\tau$  such that  $\tau \in \left(\left(\frac{1-\lambda}{1+\lambda}\right), 1\right)$ , its expected tax revenue can is

$$\Omega = (1 - \lambda)\tau + \lambda\tau [1 - G(\tau)]$$
  
=  $(1 - \lambda)\tau + \lambda\tau \frac{(1 - \lambda)(1 - \tau)}{\lambda\tau} = (1 - \lambda)$ 

The support of the mixed strategy Nash equilibrium is convex and the probability distribution of tax rates a competing country use in the equilibrium is continuous with no probability mass anywhere on the support. Therefore a competing country earns an equal tax revenue everywhere on the support. If a country sets a tax rate equal to  $(1 - \lambda) / (1 + \lambda)$ , it attracts mobile agents from the competing country with probability 1. Thus, a country can not set a tax rate lower than the infimum of the support and do strictly better. This completes the proof of the part (*III*) of the proposition (1). Q.E.D.

**Proof of Proposition (2):** As long as  $F \ge (1 + \frac{1}{\lambda})^{-1}$  holds, changes in F or  $\lambda$  will have no effect on the expected tax revenue of a competing country in the equilibrium. The reason is that a pure strategy Nash equilibrium exists in which competing countries set a tax rate equal to 1.

If  $\lambda/\left(1+\lambda+\sqrt{1+\lambda^2}\right) < F < \left(1+\frac{1}{\lambda}\right)^{-1}$ , the expected revenue of a competing country in the equilibrium is equal to  $E(R) \equiv 1-\Psi$ , where  $\Psi$  is given by (6). Differentiating E(R) with respect to F and  $\lambda$ , respectively, we have:

$$\frac{\partial E(R)}{\partial F} = \frac{1}{2} + \frac{1}{2} \frac{F + (1+\lambda)}{\sqrt{F^2 + 2F(1-\lambda) + (1-\lambda)^2}} > 0$$
(A21)

$$\frac{\partial E(R)}{\partial \lambda} = \frac{[F - (1 - \lambda)]}{2\sqrt{F^2 + 2F(1 + \lambda) + (1 - \lambda)^2}} - \frac{1}{2} < 0.$$
(A22)

In (A21), both the numerator and the denominator are greater than zero, as  $0 < \lambda < 1$ . This implies that, the right hand side of (A21) is strictly greater than zero.

If  $F - (1 - \lambda) \leq 0$ , it is trivial that the right hand side of (A22) is negative. But if  $F - (1 - \lambda) > 0$ , the right hand side of (A22) is negative when

$$F^{2} + 2F(1+\lambda) + (1-\lambda)^{2} > F^{2} + (1-\lambda)^{2} - 2F(1-\lambda) \Rightarrow 4F > 0,$$

which is true since F > 0. If  $\Upsilon \leq F \leq \lambda / (1 + \lambda + \sqrt{1 + \lambda^2})$ , the expected tax revenue of competing country in the equilibrium is  $E(R) \equiv \frac{F}{\lambda} (\sqrt{1 + \lambda^2} + 1)$ . Differentiating E(R) with respect to F and  $\lambda$ , respectively, we have

$$\frac{\partial E(R)}{\partial F} = \frac{1}{\lambda} + \sqrt{1 + \frac{1}{\lambda^2}} > 0 \tag{A23}$$

$$\frac{\partial E(R)}{\partial \lambda} = -F\left[\frac{1}{\lambda^2} + \frac{2}{\lambda^3 + \sqrt{1 + \frac{1}{\lambda^2}}}\right] < 0.$$
(A24)

This completes the proof. Q.E.D.

**Proof of Proposition (3):** Under the preferential regime competing country set tax rates on the mobile capital base and the immobile capital base independently. It is trivial that a competing country sets maximum possible tax rate (equal to 1) on the immobile capital base and earn the tax revenue equal to  $(1 - \lambda)$ .

If we substitute  $\lambda = 1$  in the proposition (1), we have the equilibrium under the non-preferential regime when all agents are mobile. Since there is no immobile capital base, the equilibrium is same when, under the preferential regime two countries compete for the mobile capital base of size 1. Let R be the tax revenue of a competing country under the non-preferential regime when  $\lambda = 1$ . Therefore, the tax revenue of a competing country from the mobile capital base under the preferential regime is  $\lambda \cdot R$ . Thus proof of the part (I) and the part (II) of the proposition (3) are similar, respectively, to the part (I) and the part (II) of the proposition (1). Proof of the part (III) of the proposition (3) is trivial. This case is similar to a Bertrand price competition. Note that if  $\lambda = 1$ , then  $\frac{F}{(1-\lambda)} \left(\frac{1}{\lambda} + \sqrt{1 + \frac{1}{\lambda^2}}\right) = \infty$ . Thus we do not need the restriction we impose on F, under the non-preferential regime. This completes the proof. Q.E.D.

**Proof of Proposition (4):** As long as  $F \ge \tau/2$ , changes in F or  $\lambda$  does not affect the tax revenue of competing countries. This is because a pure strategy Nash equilibrium exists in which competing countries set the tax revenue equal to 1 on the mobile capital base. If  $1/(2 + \sqrt{2}) < F$ < 1/2, then the expected tax revenue of competing countries is equal to  $E(R) \equiv (1 - \lambda) + \lambda (1 - \Phi)$ , where  $\Phi$  is given by (9). Differentiating E(R) with respect to F and  $\lambda$ , respectively, we have:

$$\frac{\partial E(R)}{\partial F} = \lambda \left[ \frac{1}{2} + \frac{F+2}{2\sqrt{F^2 + 4F}} \right] > 0 \tag{A25}$$

$$\frac{\partial E(R)}{\partial \lambda} = \frac{F + \sqrt{F^2 + 4F} - 2}{2} < 0.$$
(A26)

If  $F \leq 1/(2+\sqrt{2})$ , the expected tax revenue of a competing country in the equilibrium is  $E(R) \equiv$ 

 $(1-\lambda) + \lambda F(1+\sqrt{2})$ . Differentiating E(R) with respect to F and  $\lambda$ , respectively, we have:

$$\frac{\partial E(R)}{\partial F} = \lambda \left(1 + \sqrt{2}\right) > 0 \tag{A27}$$

$$\frac{\partial E(R)}{\partial \lambda} = F\left(1+\sqrt{2}\right) - 1 < 0.$$
(A28)

This completes the proof. Q.E.D.

#### **Proof of proposition (5):** We prove the proposition (5) in steps 1-5.

Step 1 – When  $F \ge 1/2$ , competing countries earn equal tax revenues from the mobile capital base under a preferential and a non-preferential regime.

This is easy to verify. When  $F \ge 1/2$ , a pure strategy Nash equilibrium exists in which competing countries set the tax rate equal to 1 on both, the mobile and the immobile capital bases. Step 2 – When  $\left(1+\frac{1}{\lambda}\right)^{-1} \le F < 1/2$ , competing countries earn higher tax revenues from the mobile capital base under a non-preferential regime when compared to a preferential regime.

In this case a pure strategy Nash equilibrium exists under the non-preferential regime, in which competing countries set the tax rate equal to 1. To the contrary, a pure strategy Nash equilibrium does not exist under the preferential regime. In the mixed strategy Nash equilibrium, competing countries set tax rates less than 1 with strictly positive probability. Thus the tax revenue from the mobile capital base is less under the preferential regime.

Step 3 – When  $1/(2+\sqrt{2}) \leq F < (1+\frac{1}{\lambda})^{-1}$ , competing countries earn higher tax revenues from the mobile capital base under a non-preferential regime when compared to a preferential regime.

In this case the tax revenues of a competing country from the mobile capital base are  $R_{np}^i \equiv \lambda \left\{ \frac{1}{2} \sqrt{F^2 + 2F (1 + \lambda) + (1 - \lambda)^2} + \frac{1}{2} (1 - \lambda) + \frac{F}{2} \right\}$  and  $R_p^i \equiv \lambda \left( \frac{F}{2} + \frac{1}{2} \sqrt{F^2 + 4F} \right)$ , respectively, under the non-preferential and the preferential regime. Note that  $(1 + \frac{1}{\lambda})^{-1} > 1/(2 + \sqrt{2}) \Rightarrow \lambda > \sqrt{2} - 1$ . When  $\lambda \leq \sqrt{2} - 1$ , a pure strategy Nash equilibrium exists under the non-preferential regime in which competing countries set the tax rate equal to 1, while a pure strategy Nash equilibrium does not exist under the preferential regime and in equilibrium, a competing country sets tax rates less than 1 with strictly positive probability. Thus, it is trivial that when  $\lambda \leq \sqrt{2} - 1$ , the expected tax revenue of a competing country from the mobile capital base is higher under the non-preferential regime. Let  $\zeta = R_{np}^i - R_p^i$  when  $\lambda > \sqrt{2} - 1$ . Hence, we have:

$$\zeta = \lambda \left\{ \frac{1}{2} (1 - \lambda) + \frac{F}{2} + \frac{1}{2} \sqrt{F^2 + 2F (1 + \lambda) + (1 - \lambda)^2} \right\} - \lambda \left( \frac{F}{2} + \frac{1}{2} \sqrt{F^2 + 4F} \right)$$
  

$$\Rightarrow \frac{\zeta}{\lambda} = \left\{ \frac{1}{2} (1 - \lambda) + \frac{F}{2} + \frac{1}{2} \sqrt{F^2 + 2F (1 + \lambda) + (1 - \lambda)^2} \right\} - \left( \frac{F}{2} + \frac{1}{2} \sqrt{F^2 + 4F} \right) (A29)$$

Given  $\lambda < 1$ ,  $\frac{\zeta}{\lambda}$  is decreasing in F. Hence, for  $F \in \left(1/\left(2+\sqrt{2}\right), \left(1+\frac{1}{\lambda}\right)^{-1}\right)$ , we find the

infimum of  $\frac{\zeta}{\lambda}$  by substituting  $F = \left(1 + \frac{1}{\lambda}\right)^{-1}$  in (A29).

$$\inf \frac{\zeta}{\lambda} \equiv \zeta^* = \frac{1}{2(1+\lambda)} \left\{ \sqrt{\left(\lambda^2 + \lambda + 1\right)^2} - \sqrt{\lambda(5\lambda+4)} + (1+\lambda)(1-\lambda) \right\}$$
  

$$\Rightarrow \zeta^* 2(1+\lambda) = \left(\lambda^2 + \lambda + 1\right) + (1+\lambda)(1-\lambda) - \sqrt{\lambda(5\lambda+4)}$$
  

$$= 2 + \lambda - \sqrt{\lambda(5\lambda+4)} > 0$$
(A30)

 $\mathrm{From}^{25}$  (A29) and (A30), it is clear that, a competing country earns higher tax revenues from the mobile capital base under the non-preferential regime when compared to the preferential regime.

Step 4 – When  $\frac{\lambda}{1+\lambda+\sqrt{1+\lambda^2}} < F \le 1/(2+\sqrt{2})$ , competing countries earn higher tax revenues from the mobile capital base under a non-preferential regime when compared to a preferential regime.

In this case a competing country earns  $R_{np}^i \equiv \lambda \left\{ \frac{1}{2} \left(1 - \lambda\right) + \frac{F}{2} + \frac{1}{2} \sqrt{F^2 + 2F \left(1 + \lambda\right) + \left(1 - \lambda\right)^2} \right\}$ and  $R_p^i \equiv \lambda F(1+\sqrt{2})$ , respectively, under the non-preferential and the preferential regime. As above let  $\zeta = R_{np}^i - R_p^i$ . Thus, we have:

$$2\frac{\zeta}{\lambda} = (1-\lambda) + F + \sqrt{F^2 + 2F(1+\lambda) + (1-\lambda)^2} - 2.F(1+\sqrt{2})$$
  
$$\zeta^* \equiv 2\frac{\zeta}{F\lambda} = \frac{(1-\lambda)}{F} + \sqrt{1 + \frac{2}{F}(1+\lambda) + \frac{1}{F^2}(1-\lambda)^2} - 1 - 2(\sqrt{2}).$$
(A31)

We can see that  $\zeta^*$  is decreasing in F. Thus for  $F \in \left(\left(1 + \frac{1}{\lambda} + \sqrt{1 + \frac{1}{\lambda^2}}\right)^{-1}, 1/(2 + \sqrt{2})\right)$ , we find the infimum of  $\zeta^*$  by substituting  $F = \left(1/\left(2+\sqrt{2}\right)\right)$  in (A31).

$$\inf \zeta^* = \sqrt{4\sqrt{2}\lambda^2 - 8\lambda + 6\lambda^2 - 6\sqrt{2}\lambda + 6\sqrt{2} + 11} - \sqrt{2}\lambda - 2\lambda - \sqrt{2} + 11$$

Since<sup>26</sup> inf  $\zeta^* > 0$ , it is clear from (A31) that, the expected tax revenue of a competing country from the mobile capital base is higher under the non-preferential regime compared to the preferential regime.

Step 5 – When  $\Upsilon \leq F \leq \frac{\lambda}{1+\lambda+\sqrt{1+\lambda^2}}$ , competing countries earn higher tax revenues from the mobile capital base under a non-preferential regime when compared to a preferential regime.

In this case a competing country earns the tax revenues equal to  $\lambda F\left(\frac{1}{\lambda} + \sqrt{1 + \frac{1}{\lambda^2}}\right)$  and  $\lambda F(1+\sqrt{2})$ , respectively, under the non-preferential and the preferential regime. Let  $\zeta \equiv \lambda F(\frac{1}{\lambda} + \sqrt{1+\frac{1}{\lambda^2}} - 1 - \sqrt{2})$ . Hence, we have  $\frac{\zeta}{F\lambda} = (\frac{1}{\lambda} + \sqrt{1+\frac{1}{\lambda^2}}) - (1 + \sqrt{2})$ , which is strictly positive for  $\lambda \in (0, 1)$ . From steps 1 – 5, proof of the proposition (5) is clear. Q.E.D.

**Proof of Proposition (6):** Note that if  $F \ge (1+\frac{1}{\lambda})^{-1}$ , a pure strategy Nash equilibrium exists under a non-preferential regime in which a competing country sets the tax rate equal to 1,

<sup>25</sup>Note that,  $2 + \lambda - \sqrt{\lambda(5\lambda+4)} > 0 \Rightarrow 4 - 4\lambda^2 > 0$ , which is true for  $\lambda < 1$ . <sup>26</sup>Note that,  $\sqrt{4\sqrt{2}\lambda^2 - 8\lambda + 6\lambda^2 - 6\sqrt{2}\lambda + 6\sqrt{2} + 11} - \sqrt{2}\lambda - 2\lambda - \sqrt{2} + 1 > 0$  iff  $4\sqrt{2}\lambda^2 - 8\lambda + 6\lambda^2 - 6\sqrt{2}\lambda + 6\sqrt{2}\lambda + 11$ 

$$6\sqrt{2} + 11 - \left(\sqrt{2\lambda} + 2\lambda + \sqrt{2} - 1\right)^2 > 0 \Rightarrow 8(1 - \lambda)\left(\sqrt{2} + 1\right) > 0, \text{ which is true as } 0 < \lambda < 1.$$

while the tax rate on the immobile capital base is fixed at 1 under the preferential regime. Hence, in this case a competing country earns an equal tax revenue under the non-preferential and the preferential regime. To the contrary, a pure strategy Nash equilibrium does not exist under the non-preferential regime when  $F < (1 + \frac{1}{\lambda})^{-1}$ . From the proposition (1), we know that in the mixed strategy Nash equilibrium, a competing country sets a tax rate less than 1 with strictly positive probability. Hence, the tax revenue under the non-preferential regime is strictly less then  $(1 - \lambda)$ . Q.E.D.

**Proof of Proposition (7):** From the proposition (1) and (3) we know that when  $F \ge 1/2$ , a pure strategy Nash equilibrium exists under both, the preferential and the non-preferential regime, in which competing countries set the tax rate equal to 1 on both forms of capital. Similarly when F = 0, both under the preferential and the preferential regime, the expected tax revenue of competing countries is equal to  $(1 - \lambda)$ . The remaining part of the proposition (7) we show in the steps 1 - 4. In each step we state a claim followed by its proof.

Step 1 – When  $(1+\frac{1}{\lambda})^{-1} \leq F < 1/2$ , a non-preferential regime generates higher tax revenues when compared to a preferential regime.

In this case a pure strategy Nash equilibrium exists under the non-preferential regime in which competing countries set the tax rate equal to 1 and earn the tax revenue amounting to 1. To the contrary, under the preferential regime a pure strategy Nash equilibrium does not exist and in equilibrium, competing countries set tax rates less than 1 with strictly positive probability. Hence, the expected tax revenue of a competing country under the preferential regime is strictly less than 1.

Step 2 – If  $1/(2+\sqrt{2}) \leq F < (1+\frac{1}{\lambda})^{-1}$ , a non-preferential regime generates higher tax revenues when compared to a preferential regime.

First of all note that  $(1 + \frac{1}{\lambda})^{-1} > 1/(2 + \sqrt{2}) \Rightarrow \lambda > \sqrt{2} - 1$ . In this case the expected tax revenues of a competing country are  $R_{np}^i \equiv 1/2(1 - \lambda) + F/2 + 1/2\sqrt{F^2 + 2F(1 + \lambda) + (1 - \lambda)^2}$  and  $R_p^i \equiv (1 - \lambda) + \lambda \left(F/2 + (1/2)\sqrt{F^2 + 4F}\right)$ , respectively, under the non-preferential and the preferential regime. We need to show that,  $\phi \equiv R_{np}^i - R_p^i > 0$ , or equivalently,

$$\frac{\phi}{F} \equiv -\frac{\theta}{2} (1-\lambda) + \frac{1}{2} + \frac{1}{2} \sqrt{1 + 2\theta (1+\lambda) + (\theta)^2 (1-\lambda)^2} - \lambda \left(\frac{1}{2} + \frac{1}{2} \sqrt{1+4\theta}\right) \ge 0$$

where,  $\theta = 1/F$ . Note that,  $F\left(1 + \frac{1}{\lambda}\right) < 1 \leq F\left(2 + \sqrt{2}\right) \Leftrightarrow \left(1 + \frac{1}{\lambda}\right) < \theta \leq \left(2 + \sqrt{2}\right)$ . We solve equation  $\phi/F = 0$  for  $\theta$ , and denote two solutions of the equation as  $\theta'$  and  $\theta''$ , where

$$\begin{array}{ll} \theta' &=& \left\{ 2\lambda \left( 2\lambda + 1/\lambda \left( 1 + \lambda + \sqrt{1 + 4\lambda^2} \right) + \sqrt{1 + 4\lambda^2} \right) \right\} \Rightarrow \theta' > 4 \mbox{ since, } 0 < \lambda < 1. \\ \theta'' &=& \left\{ 1/2\lambda \left( 2\lambda + 1/\lambda \left( 1 + \lambda - \sqrt{1 + 4\lambda^2} \right) - \sqrt{1 + 4\lambda^2} \right) \right\}. \\ &<& 1 + 1/2\lambda^2 + 1/2\lambda - 1/2\lambda - 1/2\lambda^2 \mbox{ since, } \sqrt{1 + 4\lambda^2}/2\lambda > 1/2\lambda \\ &\Rightarrow& \theta'' < 1 \Rightarrow \theta'' < \left( 1 + \frac{1}{\lambda} \right). \end{array}$$

It is known from the step 1 that, the expected tax revenue of a competing country under the nonpreferential regime is strictly higher when compared to the preferential regime when  $\theta = (1 + \frac{1}{\lambda})$ . Because the expected tax revenue of a competing country is continuous in the equilibrium, it must be true that the expected tax revenue of a competing country is higher under the nonpreferential regime when  $\theta = (1 + \frac{1}{\lambda}) + \varepsilon$  for some very small  $\varepsilon > 0$ . Also, there is no  $\theta$  s.t.  $(1 + \frac{1}{\lambda}) < \theta \leq (2 + \sqrt{2})$ , which is a solution to the equation  $\phi/F = 0$ . Hence, we show that the expected tax revenue of a competing country is higher under the non-preferential regime.

Step 3 – When  $\lambda / (1 + \lambda + \sqrt{1 + \lambda^2}) \leq F < 1/(2 + \sqrt{2})$ , a non-preferential regime generates higher tax revenues when compared to a preferential regime.

First of all note that,  $(2 + \sqrt{2}) < (\lambda + 1 + \sqrt{\lambda^2 + 1}) / \lambda$  for  $\forall \lambda \ s.t. \ 0 < \lambda < 1$ . As before let  $\phi \equiv R_{np}^i - R_p^i$ , where  $R_{np}^i$  and  $R_p^i$  are expected tax revenues of a competing countries, respectively, under the non-preferential and preferential regime when  $1/(2 + \sqrt{2}) > F \ge \lambda / (1 + \lambda + \sqrt{1 + \lambda^2})$ .

$$R_{np}^{i} = (1/2) (1 - \lambda) + (1/2) F + (1/2) \sqrt{F^{2} + 2F (1 + \lambda) + (1 - \lambda)^{2}},$$
  

$$R_{p}^{i} = (1 - \lambda) + F (1 + \sqrt{2}) \lambda.$$

Now,  $\phi > 0$  if

$$\begin{split} \sqrt{1+2\theta\left(1+\lambda\right)+\theta^2\left(1-\lambda\right)^2} &\geq \theta\left(1-\lambda\right)+2\left(1+\sqrt{2}\right)\lambda-1>0\\ \Rightarrow \theta &> \frac{\lambda\left(1+\sqrt{2}\right)\left[\lambda\left(1+\sqrt{2}\right)-1\right]}{1-\lambda\left(1-\lambda\right)\left(1+\sqrt{2}\right)}, \end{split}$$

where  $\theta = \frac{1}{F}$ . Note that  $1 - \lambda (1 - \lambda) (1 + \sqrt{2}) > 0$  for all  $\lambda$  s.t.  $0 < \lambda < 1$ . Thus, when  $\lambda \le \sqrt{2} - 1$ , the numerator is less than zero and the denominator is greater than zero. Thus the condition holds trivially. Now for  $\lambda > \sqrt{2} - 1$ , both the numerator and the denominator are positive. Thus, to prove the claim we only need to show that,

$$\frac{\lambda\left(1+\sqrt{2}\right)\left[\lambda\left(1+\sqrt{2}\right)-1\right]}{1-\lambda\left(1-\lambda\right)\left(1+\sqrt{2}\right)} < 2+\sqrt{2} \Rightarrow \lambda-\lambda^2 - \frac{2+\sqrt{2}}{\left(1+\sqrt{2}\right)^2} < 0 \tag{A32}$$

Now let  $\Phi = \lambda - \lambda^2 - (2 + \sqrt{2}) / (1 + \sqrt{2})^2$ . We can see that  $\lambda^{27} \Phi(\lambda)$  is a strictly concave function of  $\lambda$ .

$$\max \Phi(\lambda) = \frac{1}{2} - \frac{1}{4} - \frac{2 + \sqrt{2}}{\left(1 + \sqrt{2}\right)^2} < 0.$$
(A33)

From (A32) and (A33) it is clear that  $\phi > 0$ .

Step 4 – From steps 1–3 it is clear that as long as  $\lambda / \left(1 + \lambda + \sqrt{1 + \lambda^2}\right) < F$ , a competing country earns a higher tax revenue under the non-preferential regime when compared to the preferential regime. The relation is strict if  $\lambda / \left(1 + \lambda + \sqrt{1 + \lambda^2}\right) < F < 1/2$ . Now to prove the proposition (7), we compare the tax revenues of a competing country when  $\lambda / \left(1 + \lambda + \sqrt{1 + \lambda^2}\right) \geq F$ .

When  $\lambda/\left(1+\lambda+\sqrt{1+\lambda^2}\right) \geq F \geq (1-\lambda)\left(\frac{1}{\lambda}+\sqrt{1+\frac{1}{\lambda^2}}\right)^{-1}$ , a competing country earns tax revenues (in expected term) amounting to  $R_{np}^i \equiv F\left(\frac{1}{\lambda}+\sqrt{1+\frac{1}{\lambda^2}}\right)$  and  $R_p^i \equiv (1-\lambda)1+F\left(1+\sqrt{2}\right)\lambda$ , respectively, under the non-preferential and the preferential regime. As before, let  $\phi \equiv R_{np}^i - R_p^i$ . Thus, we have

$$\phi = F\left(\frac{1}{\lambda} + \sqrt{1 + \frac{1}{\lambda^2}}\right) - (1 - \lambda)1 - F\left(1 + \sqrt{2}\right)\lambda \ge 0$$
  
$$\Leftrightarrow 1 \le \frac{1}{(1 - \lambda)} \left[F\left(\frac{1}{\lambda} + \sqrt{1 + \frac{1}{\lambda^2}}\right) - \lambda F\left(1 + \sqrt{2}\right)\right].$$
(A34)

Rearranging (A34) we get,  $\phi < 0$  if  $F^1 \leq F \leq F^2$ , and  $\phi > 0$  when  $1/2 > F > F^2$ , where  $F^1$ and  $F^2$  are given by (10) and (11) respectively. Thus when  $F^1 \leq F \leq F^2$ , a competing country earns higher tax revenues under the preferential regime and to the contrary when  $1/2 > F > F^2$ , it earns higher tax revenues under the non-preferential regime. This completes the proof. Q.E.D.

 ${}^{27}\frac{\partial^2 \Phi}{\partial \lambda^2} = -2 < 0.$ 

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