On Polarized Prices and Costly Sequential Search

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Abstract: This paper presents a homogenous goods duopoly model of costly sequential consumer search with three classes of consumers: costless searchers; moderately costly searchers; and consumers for whom search costs are extremely high–higher than the value they attach to the good. Under certain conditions, the mixed-strategy Nash equilibrium price distribution is one where low and high, but never moderate, prices are charged. In equilibrium, free searchers will always search for both prices, very costly searchers never will, and moderately costly searchers will engage in actual search with positive probability. Interestingly, the existence of consumers who do not themselves search for prices allows for the introduction of an equilibrium where costly search does occur.

1 Introduction

For nearly half a century, economists have been studying the role of costly consumer search on firm and customer behavior. The existence of search costs allows for the price dispersion that is present in many markets. Further, we can readily observe that people do engage in search behavior, comparing prices and features across products and firms. In this paper, we study the consequences of costly search in a market where search costs are prohibitively high for a fraction of consumers, moderate for others, and free for the rest.

Diamond (1971) presents a model where all consumers must pay a positive, albeit potentially miniscule, search cost to obtain prices. The result is the Diamond Paradox–if all consumers must pay even a vanishingly small search cost, then no search will occur and the equilibrium outcome will be the monopoly price–an outcome which is not borne out by simple observation. Of the many solutions to this puzzle, the most appealing is offered by Stahl (1989). The key difference between the Stahl and Diamond models is that Stahl allows for some fraction of consumers to search costlessly, while the others incur a positive search

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cost. The existence of these free searchers serves to mitigate the market power found by Diamond, creating instead a market which exhibits price dispersion in equilibrium. Competition between firms will bid down prices to a level low enough such that no costly searcher will ever choose to pay the search cost to obtain pricing information.

This paper extends the Stahl (1989) model by adding a third class of consumers–those for whom search is quite costly: so costly, in fact, that they will never choose to engage in any price comparison. For these consumers, the cost of searching outweighs the valuation they attach to the good. This is a simple extension of the Stahl assumption that free searchers exist: if costless searchers can be conceptualized as people who enjoy shopping, are internet savvy, or have a negligible opportunity cost of time, very costly searchers can be envisioned in an opposing manner. Perhaps these consumers hate shopping, are Luddites, or value their time very highly. Moderately costly searchers fall between these two extremes: perhaps they have Internet access, but are not as efficient as free searchers at using it. This extension yields two important results:

1. *Polarized pricing*: the equilibrium distribution of prices is polarized in nature. Firms will charge low or high, but never moderate prices. This distribution is qualitatively quite different from that described by Stahl (1989), where only relatively low prices are charged. This polarized pricing equilibrium serves to explain the phenomenon described in Varian (1980, p. 658):

   > [C]hains such as Sears and Roebuck and Montgomery Ward sell appliances at their regular price much of the time, but often have sales when the price is reduced by as much as 25 percent. However, we rarely observe them selling an appliance at an intermediate price.

2. *Actual search*: In this model, price competition between firms does not result in prices low enough to rule out actual search on the part of costly searchers. If a consumer observes a price drawn from the "high" portion of the price distribution, then she will incur a search cost to obtain an additional price quotation. Thus, the somewhat ironic result of this paper is that the existence of people who will themselves never search (the very costly searchers) brings about an equilibrium where costly search occurs with strictly positive probability.

Variations on both of these results have been found in the nonsequential search literature, but are novel within the framework of sequential search. Salop and Stiglitz (1977) model searching as purchasing a newspaper with complete information–consumers are either uninformed or perfectly informed, which is accomplished by incurring a single search cost. This model generate several equilibria, one of which is a two price equilibrium where the competitive price and a higher price are charged, but never any intermediate prices.[1]

[1] Though both the two-price equilibrium (TPE) result and the result presented in this

Similarly, the result of actual search–that is, acquisition of at least two prices by costly searchers–has been shown in models of nonsequential search. Burdett and Judd (1983) present a model of nonsequential price search; in equilibrium, customers may pay to observe more than one price. Janssen and Moraga-Gonzàlez (2004) and Janssen, Moraga-Gonzàlez, and Wildenbeest (2005a) each present a variation of the Burdett and Judd nonsequential[2] search model by relaxing the assumption that the first price quotation is free. In these models, they find that several equilibria arise, most notably (for our purposes) the high search intensity equilibrium, where less-informed consumers randomize between searching for one and two prices.

The very costly searchers examined in this paper behave as captive consumers–though they are capable of searching for additional price information that would allow them to purchase beyond their randomly-selected firm, it is never optimal to do so. Thus, this model can be alternatively conceptualized as a captive markets model where some consumers are free switchers, some are captive, and some can switch between firms, but only at a cost. The classic paper by Varian (1980) examines a model with informed and uninformed consumers, the former of whom can find the cheapest price freely and instantaneously, whereas the latter are captive to a randomly-selected firm. Firms then select prices from a price-dispersed equilibrium.[3]

The rest of the paper is organized as follows. Section 2 will formally introduce the model. Section 3 will establish the equilibrium price distributions, which will be discussed in Section 4. Section 5 will conclude.

2 The Model

Consider a homogenous goods duopoly where consumers are *ex ante* uninformed of prices, and must engage in sequential search to obtain this information. The search technology is such that obtaining the first price quotation is free, but consumers must incur a cost to obtain the second price; thus the term "search cost" refers throughout to the cost of searching for the second price. All consumers have identical unit demand, and will buy exactly one unit if the price

paper exhibit polarized prices, these models differ in two important respects. First, in the TPE firms earn zero expected profits, whereas here, expected profits are bounded above zero. Additionally, the polarized pricing equilibrium distribution studied here has two disjoint supports, each of which has positive measure, whereas the TPE is comprised only of two mass points.

[2] Janssen, Moraga-Gonzàlez and Wildenbeest (2004, 2005b) study a variation of the Stahl (sequential) search model with costly initial search and do not find that actual search of two or more prices occurs on the part of costly searchers.

[3] Narasimhan (1988) examines a duopoly model where a fraction of consumers are loyal to one brand or the other, and other consumers will switch between brands. Under one assumption of the Narasimhan model, the switchers are indifferent between brands, and care only about prices; in this paper, when the measure of moderately costly searchers is zero, the Narasimhan model is obtained. Free searchers can be conceptualized as free switchers, and very costly searchers are brand loyalists. Deneckere, Kovenock, and Lee (1992) adapt the Narasimhan model to discuss issues of price leadership.

charged, p, is no greater than valuation $\theta > 0$.[4] Consumers are exogenously heterogenous in search costs, and otherwise identical. Consumers have perfect recall and can costlessly purchase the good from any firm whose price they have observed. The three types of consumers are as follows:

- A fraction n_H of consumers can search for the second price only at an extremely high cost, $s_H > \theta$. These consumers are called *very costly searchers.*
- A fraction n_Z of consumers have zero search cost. Call these consumers *free searchers*
- The remaining fraction n_C of consumers have a search cost of s, where $s \in (0, \theta)$. These consumers are called *costly searchers.*

The measure of consumers is normalized to one, so that $n_H + n_Z + n_C = 1$. Firms are aware of the size of each segment of consumers, but are unable to engage in price discrimination. Marginal cost of production is constant; we will normalize this cost to zero. The game proceeds as follows: in the first stage, firms set prices, and in the second stage, consumers search for prices and possibly buy the good.

3 Equilibrium

3.1 Consumer Behavior

In the second stage, consumers shop (i.e. engage in price search) for the good and buy it so long as its price is no greater than the value attached to the good, θ. First, note that the free searches will always obtain both prices, as it is costless to do so. Thus, these n_Z consumers will always buy from the cheaper firm. Second, note that the n_H very costly searchers will never choose to search, because incurring a search cost of $s_H > \theta$ will always drive the functional price of the good above valuation θ. As no firm will ever charge $p > \theta$, these consumers will never engage in search. Here we study a symmetric Nash equilibrium. In such an equilibrium, nonsearchers will randomize their initial search evenly between the two firms, and each firm will receive a fraction $\frac{n_H}{2}$ of the prohibitively-costly searching segment.

Lastly, consider the n_C moderately costly searchers. These consumers will, again, always obtain the first (free) price quotation, randomizing equally between the two firms. Furthermore, they will choose to obtain a second price quotation at cost $s > 0$ if the expected benefit of doing so, in the form of a lower price, is greater than s. Each costly searcher will therefore establish a reservation price such that if the price obtained by their first price sample is

[4] Unit demand, in addition to lending computational tractability, also removes quantity effects from social welfare analysis, so that all changes in social welfare can be measured as changes in price.

greater than the reservation price, then he will engage in costly search to obtain the rival firm's price. In principle, these reservation prices may be different for each firm, but in the case of symmetric Nash equilibrium studied here, each firm will share a common reservation price, which is denoted ρ.

Due to the unit demand structure, expected consumer surplus (not accounting for search costs incurred) is given simply by

$$ECS = \theta - E(p)$$

We define the reservation price, ρ by

$$\rho = ECS - s \tag{1}$$

Thus, the optimal search rule for moderately costly searchers is to always search for the first price (as to do so is costless) and to search for the price of the second good if the price obtained at the first firm is greater than the reservation price ρ. These consumers will stop searching and purchase the good after the first firm if the price quoted, $p \leq \rho$.

3.2 Characterization of Equilibrium

Note that this paper presents a straightforward extension of the model presented in Stahl (1989). If $n_H = 0$, then this model collapses exactly to the Stahl case. We seek to show that when there are consumers who search at a very high cost, an equilibrium which is qualitatively different from that presented in Stahl may arise. One restriction on s is required to ensure that an equilibrium which is qualitatively different from the Stahl case arises.[5] This condition is $s < s^a$ where s^a is given by the equation

$$s^a = \left[\ln\left(\frac{1-n_Z}{1+n_Z}\right)\cdot\frac{n_H}{2n_Z} + \frac{n_H}{1-n_Z}\right]\cdot\theta \tag{2}$$

Let $g(x,s)$ be the function defined by

$$\begin{aligned} g(x,s) \;=\; & \frac{n_H\theta}{2n_Z}\cdot\left\{\ln(x) - \ln\left(\frac{\theta x n_H\left(n_H-n_Z-1\right)}{2xn_Z\left(n_H-2\right)+\theta n_H\left(n_H+n_Z-1\right)}\right)\right. \\ & \left. + \left(\frac{n_Z}{1-n_H}\right)\left[\ln(\theta) - \ln\left(\frac{n_H x\theta(2n_Z+n_C)}{2n_H\theta\left(n_Z+n_C\right)-xn_C\left(n_H+2n_Z+2n_C\right)}\right)\right]\right\} - s \end{aligned} \tag{3}$$

We define a critical value of s, denoted $s^\star$:

$$\begin{aligned} s^\star \;=\; & -\frac{1}{2}\left(\frac{1}{n_Z\left(n_H-1\right)}\right)\left\{\theta n_H\left(n_H-1\right)\cdot\ln\left(\frac{-4n_Z+3n_Hn_Z-n_H+n_H^2}{n_H\left(n_H-n_Z-1\right)}\right)\right. \\ & \left. -n_Hn_Z\ln\left(\frac{2-5n_H-2n_Z+3n_H^2+n_Hn_Z}{n_H\left(n_H-n_Z-1\right)}\right)+2n_Z(1-n_H)\right\} \end{aligned} \tag{4}$$

[5] If the following equation does not hold, the equilibrium is similar to the Stahl equilibrium, where only prices below the reservation price are charged in equilibrium. This case is presented in the appendix.

We assume that $s^\star < s^a$; in the appendix we present parameter values for which this condition holds–for example, if half of all consumers are very costly searchers, and the remaining consumers are evenly split between free searchers and moderately costly searchers (that is, $n_H = \frac{1}{2}, n_Z = n_C = \frac{1}{4}$), then it will be the case that $s^\star < s^a$ for $\theta > 1.55$.

We now establish a preliminary result, which is proven in the appendix.

Lemma 1 *For $s > s^\star$ $\exists\, x \in (0, \theta)$ such that $g(x) = x$.*

Now let us present the main result:

Proposition 2 (Main Result) *Suppose that $s < s^a$ where s^a is given by equation (2) holds. Then there exists a unique symmetric mixed strategy Nash equilibrium where the support of the price distribution consists of two disjoint intervals $[b, \rho] \cup [\varphi, \theta]$ where $b < \rho < \varphi < \theta$, and moderately costly searchers engage in actual search whenever the first price observed is above ρ, the reservation price.*
In particular, b and φ are defined respectively by

$$b = \frac{\theta \rho n_H (n_H - n_Z - 1)}{2\rho n_Z (n_H - 2) + \theta n_H (n_H + n_Z - 1)} \tag{5}$$

$$\varphi = \frac{n_H \rho \theta (2n_Z + n_C)}{2n_H \theta (n_Z + n_C) - \rho n_C (n_H + 2n_Z + 2n_C)} \tag{6}$$

and ρ is defined implicitly by the fixed point of the function given by equation (3). Further, this ρ is smaller than θ for any $s \leq s^\star$[6] *where $s^\star$ is given by equation(4).The equilibrium cumulative distribution function for prices is given by*

$$F(p) = \begin{cases} 0 \text{ for } p < b \\ \frac{1}{2pn_Z}\left[n_H (p-\theta) + 2p(n_C + n_Z) - pn_C \cdot \frac{n_H(\rho-\theta)+2\rho(n_C+n_Z)}{(2n_Z+n_C)\rho}\right] \text{ for } b \leq p \leq \rho \\ \frac{n_H(\rho-\theta)+2\rho(n_C+n_Z)}{(2n_Z+n_C)\rho} \text{ for } \rho < p < \varphi \\ \frac{n_H(p-\theta)}{2p(1-n_H)} + 1 \text{ for } \varphi \leq p \leq \theta \end{cases}$$

Further, in equilibrium, consumers behave in the following manner:

- *Free searchers will search for both prices and purchase from the lower-priced firm*
- *Very costly searchers will obtain only one price quotation, randomizing equally between both firms, and will buy from the sampled firm.*
- *Moderately costly searchers will obtain the first price; if this price is revealed to be above ρ, then they will search again, and buy the good from the cheaper*

[6] If $s > s^\star$ so that $\rho > \theta$, then the equilibrium price distribution is given by

$$F(p) = \begin{cases} 0 \text{ for } p < b \\ \frac{1}{2pn_Z}\left[n_H (p-\theta) + 2p(n_C + n_Z) - pn_C \cdot \frac{n_H(\rho-\theta)+2\rho(n_C+n_Z)}{(2n_Z+n_C)\rho}\right] \text{ for } b \leq p \leq \theta \end{cases}$$

as prices greater than θ are never charged.

firm. If the first price is revealed to be below ρ, then they will buy the good from the first (and only) firm sampled.
Finally, because costly search is undertaken with positive probability, the expected search cost is positive. Namely, expected search cost is

$$\begin{aligned} E(s) &= s(1-F(\rho)) \\ &= s\frac{[\rho\,(n_Z-1)+\theta n_H]}{\rho\,(1+n_Z-n_H)} > 0 \end{aligned}$$

Proof: First, note that given the price distribution described in the Proposition, consumers are behaving optimally. Free searchers always obtain a second price; very costly searchers never do. Moderately costly searchers do when the cost of searching s, is smaller than the expected benefit of finding a lower price.

To verify the optimality of the firm behavior, first note that in any mixed price equilibrium, all prices charged with positive probability must yield the same expected payoff. If a firm charges $p \in (\rho, \theta]$, then it can expect to sell to its $\frac{n_H}{2}$ very-costly searchers with full probability, and will sell to all of the remaining $n_Z + n_C$ consumers if its price is the lowest[7], which occurs with probability $(1 - F(p))$. Therefore, expected profit is given by

$$E\pi(p|\rho < p \leq \theta) = \left\{\frac{n_H}{2} + [1-F(p)]\,(n_Z+n_C)\right\} \cdot p \tag{7}$$

If the firm instead chooses to set a price below ρ, then it will continue to sell to its n very-costly searchers, as well as its half of the moderately-costly searchers (who will not search when faced with an initial price below ρ) with full probability. It will sell to the free searchers, n_Z, if its price is the lowest and will sell to its rival's half of the costly searchers if the price charged by its rival is greater than ρ, which occurs with probability $(1 - F(\rho))$. Therefore, expected profit will be given by

$$E\pi(p|b \leq p \leq \rho) = \left\{\frac{n_H}{2} + \frac{n_C}{2} + [1-F(p)]\,n_Z + [1-F(\rho)]\,\frac{n_C}{2}\right\} \cdot p \tag{8}$$

Taking the limits of equations (7) and (8) as $p \to \rho$, we find

$$\begin{aligned} \lim_{p\to\rho} E\pi(p|\rho &< p \leq \theta) = \left\{\frac{n_H}{2} + [1-F(\rho)]\,(n_Z+n_C)\right\} \cdot \rho & (9) \\ \lim_{p\to\rho} E\pi(p|b &\leq p \leq \rho) = \left\{\frac{n_H}{2} + \frac{n_C}{2} + [1-F(\rho)]\left(n_Z + \frac{n_C}{2}\right)\right\} \cdot \rho & (10) \end{aligned}$$

where (9) $\neq$ (10) . This confirms the description of the "shape" of the equilibrium–that the Nash equilibrium distribution of prices, F, will have a support with two disjoint sections, with the disjoint occurring at ρ.

[7] If $\rho < p_i$ and $p_i < p_j$, it is clear that $\rho < p_j$, so that if, when firm i charges a price greater than ρ, it is the cheapest price, then firm j has also charged greater than ρ. Therefore, all of firm $j's$ moderately-costly searchers are also searching.

Having thus established the shape of the equilibrium price distribution, let us now formally derive it. For both θ and ρ to be charged with positive probability, it must be the case that expected profits are equalized at $p = \rho$ and $p = \theta$, as well as at any other price which is in the support of the distribution of F.

If the firm chooses to set a price above ρ, its expected profit is given by equation (7). If a firm charges $p = \theta$, it will be undercut with full probability, and will sell to only its very costly searchers, having induced search on the part of its moderately-costly searchers. In this case, profit will simply be $E\pi(\theta) = \frac{n_H \theta}{2}$. Equating these expected profits and solving for F yields the following distribution of prices for $\rho < p \leq \theta$:

$$F(p) = \frac{n_H (p - \theta)}{2p (1 - n_H)} + 1 \text{ for } p \in (\rho, \theta] \tag{11}$$

where, in order for $F(p)$ to be a valid distribution function, it must only be defined on $F(p) \in [0, 1]$. Therefore, we can see that the above function is only a valid distribution when:

$$-1 \leq \frac{n_H (p - \theta)}{2p (1 - n_H)} \leq 0 \tag{12}$$

Which is satisfied for all p[8]. Thus, $F(p)$ is a valid distribution function for all values of p, and equation (11) describes the portion of the Nash equilibrium distribution price distribution which lies above ρ.

We shall now solve for the portion below ρ. Again, for all prices in the support of F, expected profits must be equalized. Therefore, (8) can also be equated with $E\pi(\theta) = n\theta$. Solving for F yields:

$$F(p) = \frac{1}{2pn_Z} \left[n_H (p - \theta) + 2p (n_C + n_Z) - pn_C \cdot F(\rho) \right] \text{ for } p \in [b, \rho] \tag{13}$$

where $F(\rho)$ is a constant[9]:

$$F(\rho) = \frac{n_H(\rho - \theta) + 2\rho(n_C + n_Z)}{(2n_Z + n_C)\,\rho}$$

Inserting this $F(\rho)$ into equation (13) completes the portion of the price distribution which lies below ρ:

$$F(p) = \frac{1}{2pn_Z} \left[n_H (p - \theta) + 2p (n_C + n_Z) - pn_C \cdot \frac{n_H(\rho - \theta) + 2\rho(n_C + n_Z)}{(2n_Z + n_C)\,\rho} \right] \text{for } p \in [b, \rho]$$

[8] Proof is available from the author upon request.

[9] $F(\rho)$ is found by solving equation (13) using $p = \rho$

$$F(\rho) = 1 - \frac{1}{2n_Z} \left\{ n_H \left(\frac{\theta - \rho}{\rho} \right) - n_C \cdot (2 - F(\rho)) \right\}$$

So the complete, piecewise distribution of prices is given by:

$$F(p)=\begin{cases} 0 \text{ for } p<b \\ \frac{1}{2pn_Z}\left[n_H\left(p-\theta\right)+2p\left(n_C+n_Z\right)-pn_C\cdot\frac{n_H(\rho-\theta)+2\rho(n_C+n_Z)}{(2n_Z+n_C)\rho}\right] \text{ for } b\leq p\leq\rho \\ \frac{n_H(\rho-\theta)+2\rho(n_C+n_Z)}{(2n_Z+n_C)\rho} \text{ for } \rho<p<\varphi \\ \frac{n_H(p-\theta)}{2p(1-n_H)}+1 \text{ for } \varphi\leq p\leq\theta \end{cases} \tag{14}$$

Where φ represents the lowest price above ρ that a firm is willing to charge in equilibrium. In order to complete the characterization of the Nash equilibrium, we must solve for φ, b, and ρ. ρ is the fixed point of $g(\rho)$ given by equation (3); by Lemma 1, this fixed point exists. We can use the fact that $F(\varphi)=F(\rho)$ to solve for φ :

$$\begin{aligned} \frac{n_H\left(\varphi-\theta\right)}{2\varphi\left(1-n_H\right)}+1 &= \frac{2\rho-n_H(\theta+\rho)}{\left(2n_Z+n_C\right)\rho} \\ \varphi &= \frac{n_H\rho\theta(2n_Z+n_C)}{2n_H\theta\left(n_Z+n_C\right)-\rho n_C\left(n_H+2n_Z+2n_C\right)} \end{aligned} \tag{15}$$

We solve for b by setting

$$\begin{aligned} F(p) &= 0 \\ b &= \frac{\theta\rho n_H\left(n_H-n_Z-1\right)}{2\rho n_Z\left(n_H-2\right)+\theta n_H\left(n_H+n_Z-1\right)} \end{aligned}$$

.

Again, ρ is given by the solution to $\rho-E(p)-s=0$, where expected price may be written as:

$$\begin{aligned} E(p) &= \int_b^\rho p\cdot f(p)dp+\int_\varphi^\theta p\cdot f(p)dp \\ &= \frac{n_H\theta}{2n_Z}\cdot\left[\ln\left(\frac{\rho}{b}\right)+\left(\frac{n_Z}{1-n_H}\right)\ln\left(\frac{\theta}{\varphi}\right)\right] \end{aligned}$$

so that ρ is given implicitly by

$$\rho=\frac{n_H\theta}{2n_Z}\cdot\left[\ln\left(\frac{\rho}{b}\right)+\left(\frac{n_Z}{1-n_H}\right)\ln\left(\frac{\theta}{\varphi}\right)\right]-s \tag{16}$$

Thus, we have established the Nash equilibrium price distribution claimed in Proposition 1. All that remains to be shown is that expected profits are no higher at any price outside the support of F. First, note that prices above θ will result in no sales, as no consumer will buy the good at a price higher than the good's valuation; thus higher prices cannot yield a higher profit. It is similarly straightforward to claim that b is, by construction, the lowest price that the firm is willing to accept; any lower, and the firm will prefer to charge the monopoly price θ and sell only to its n very costly searchers. What remains to be shown is that it is never optimal to charge a price in the gap between ρ and φ. Consider a price $\hat{p} \in (\rho, \varphi)$. We will show that neither firm has an incentive to unilaterally deviate to such a $\hat{p}$.

If one firm charges $\hat{p}$ while its rival continues to play the price distribution above, then it can expect to sell to its share n of the very costly searchers, who will not compare prices. It will, having charged a price greater than ρ, have induced search on the part of its moderately costly searchers, and will sell to all of the remaining m consumers if its price is the lowest. Thus, its expected profit is given by

$$E\pi(\hat{p}) = \left\{ \frac{n_H}{2} + [1 - F(\hat{p})] (1 - n_H) \right\} \cdot \hat{p} \tag{17}$$

where $[1 - F(\hat{p})]$ represents the probability that the rival firm is charging a price greater than $\hat{p}$. Note that for the rival firm, which is playing according to the equilibrium price distribution described in equation (14), $[1 - F(\rho)]$ $=[1 - F(\varphi)]$ $=[1 - F(\hat{p})]$. Hence, we can see that

$$\begin{aligned} E\pi(\hat{p}) &= \left\{ \frac{n_H}{2} + [1 - F(\varphi)] (1 - n_H) \right\} \cdot \hat{p} \\ &< \left\{ \frac{n_H}{2} + [1 - F(\varphi)] (1 - n_H) \right\} \cdot \varphi \end{aligned}$$

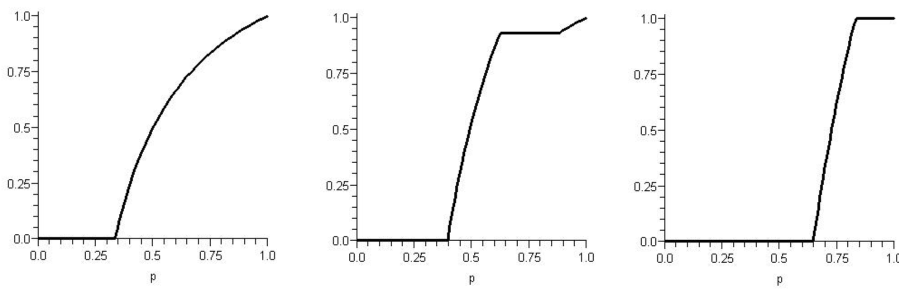
where the inequality follows from $\hat{p} < \varphi$. By charging a price $\hat{p}$ which falls in the gap between ρ and φ, the firm is reducing the price it receives without a corresponding increase in sales; hence, it lowers its expected profits. Thus, the firm does not have an incentive to deviate and charge a price outside of the support of F. **Q.E.D.**

4 Discussion

Having established that the support of the equilibrium price distribution in this model is composed of two disjoint segments with the disjoint occurring at ρ, let us examine the intuition behind this result. First, consider a market with no moderately costly searchers, so that the only consumers remaining search either costlessly or at a prohibitively high cost. Note that as the fraction of costless searchers goes to one, price will fall to marginal cost (here, zero); similarly, as the fraction of very costly searchers goes to one, the price will rise to the monopoly price. The existence of both types of consumers, as examined in Narasimhan (1988) and Deneckere, Kovenock, and Lee (1992) will lead to a mixed strategy Nash equilibrium. Here, profits will be equalized with the profits earned by selling only to the very costly searchers (who will never search) at the monopoly price, and the support of the distribution will be $[b, \theta]$ where charging any price lower than b will yield strictly lower profits than charging θ.

Now, introduce a small measure of moderately costly searchers. These consumers will set a reservation price $\rho \in [b, \theta]$. If the first price observed is no greater than ρ, the consumer will buy from the first firm sampled; otherwise,

Figure 1: NE Price Distribution as n_C increases



(a) NE Price distribution when n_C=0

(b) NE Price distribution when n_C=0.25

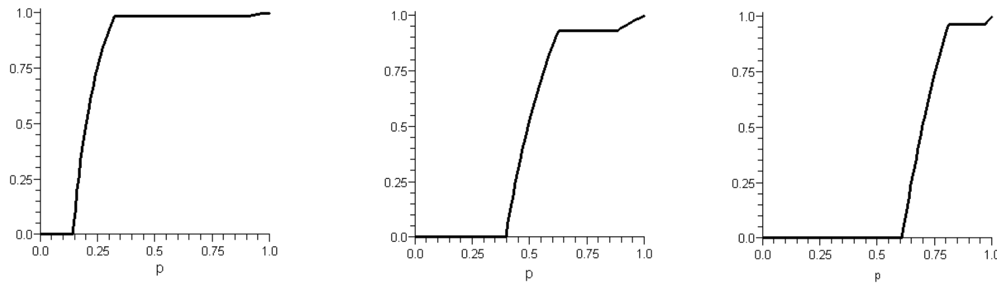
(c) NE Price distribution when n_C=0.375

the consumer will obtain a second price quotation. Given the behavior of these moderately costly searchers, firms cannot maintain their original pricing distribution as an equilibrium–if a firm charges a price slightly higher than ρ, it will have induced search among all of its moderately costly searchers, and will run the risk of losing them to its rival. The firm will be unwilling to charge prices only slightly higher than ρ, as doing so will yield only a marginal increase in per-consumer revenue, but will decrease the probability of making a sale by inducing the moderately costly searchers to engage in search. Thus, though firms may be willing to charge prices "much" higher than ρ, and as high as θ, they will be unwilling to charge prices only "slightly" higher than ρ in equilibrium.

We can see this dynamic at play in Figure (1). As shown in panel (a), when there are no moderately costly searchers, there is no gap in prices charged between ρ and φ; prices as high as θ are charged in equilibrium. When a small number of moderately costly searchers enters the market ($n_C = 0.25$), then the gap between ρ and φ appears–this is shown in panel (b), where the disjoint in the support of F is shown by the horizontal segment. Lastly, if the size of the moderately costly searching segment is large enough (drawn as $n_C = 0.375$) then Condition (a) will fail to hold, and the equilibrium will be Stahl-like, with the upper bound of the distribution at ρ, as shown in panel (c).

It is also interesting to ask: how does the market change as the size of the very costly searching market, n_H, changes? When the entire market is comprised of very costly searchers (i.e. when $n_H = 1$), this model collapses to the Bertrand model of competition, so that the only price charged in equilibrium will be the monopoly price. Similarly, when there are no very costly searchers ($n_H = 0$) the model collapses to the classic Stahl model. For small values of n_H, condition (a) will not be satisfied, and the equilibrium price distribution will be qualitatively similar to that presented in Stahl–namely prices up to ρ will be charged, but no higher; this ρ is increasing in n_H. It can also be shown that φ is decreasing in the size of the very costly searching segment. Thus, as

Figure 2: NE Price Distribution as n_H increases



(a) NE Price distribution when n_H=0.2 (b) NE Price distribution when n_H=0.5 (c) NE Price distribution when n_H=0.8

n_H increases, the gap between ρ and φ will shrink; this is consistent with the logic outlined in the previous two paragraphs. An increase in n_H will likely cause a decrease in n_C[10], the size of the moderately costly market segment. The smaller is the moderately costly segment, the smaller is the gap between prices charged. This is shown in Figure (2) and n_H increases from 0.2 to 0.5 to 0.8, and the gap between ρ and φ shrinks. Note that Figure (2) is drawn such that the free searchers and costly searchers are of equal measure: $n_Z = n_C$.

We may also ask: when is it likely that Condition (a) will be satisfied so that the above-described equilibrium will hold? Recall Condition (a):

$$\frac{s}{\theta} < \ln\left(\frac{1-n_Z}{1+n_Z}\right) \cdot \frac{n_H}{2n_Z} + \frac{n_H}{1-n_Z}$$

which may be expressed equivalently as

$$\frac{s}{\theta} < \ln\left(\frac{1-n_Z}{1+n_Z}\right) \cdot \frac{1-n_Z-n_C}{2n_Z} + \frac{1-n_Z-n_C}{1-n_Z} \tag{a'}$$

We will examine this condition as n_Z becomes very small or very large; likewise with n_H and n_C.

The RHS of Condition (a) as n_Z becomes small is given by

$$\lim_{n_Z \to 0} \ln\left(\frac{1-n_Z}{1+n_Z}\right) \cdot \frac{n_H}{2n_Z} + \frac{n_H}{1-n_Z} = 0$$

so that for any positive search cost, Condition (a) cannot hold. As the fraction of consumers who can search for free n_Z goes to zero, and we find ourselves a Diamond-type situation where all consumers must pay to search. Consequently,

[10] unless n_C is assumed to be held constant, in which case the entire increase in n_H will be offset by a decrease in n_Z

no search will ever occur, and the equilibrium outcome will be monopoly pricing. Conversely, when the number of free searchers n_Z approaches unity, then the model collapses to the Bertrand model where all consumers can perfectly observe the cheapest price, and the equilibrium price is driven down to zero.

When n_H is very small, the RHS of Condition (a) is given by

$$\lim_{n_H \to 0} \ln\left(\frac{1-n_Z}{1+n_Z}\right) \cdot \frac{n_H}{2n_Z} + \frac{n_H}{1-n_Z} = 0$$

so that Condition (a) cannot hold for any positive search cost. When there are no high-cost searchers, then the model collapses to that of Stahl, and the equilibrium result is that firms will price below the reservation level, so that no costly search will occur. When, on the other hand, n_H approaches unity, then all consumers are very costly searchers. Thus this model also collapses to the Diamond result where the only price charged in equilibrium is the monopoly price.

When n_C is very small, the RHS of Condition (a') is given by

$$\lim_{n_C \to 0} \ln\left(\frac{1-n_Z}{1+n_Z}\right) \cdot \frac{1-n_Z-n_C}{2n_Z} + \frac{1-n_Z-n_C}{1-n_Z}$$

This is presented in Figure (3), where the shaded portion of the graph represents values of $\frac{s}{\theta}$ where Condition (a') is satisfied. As the measure of free searchers, n_Z increases, it is increasingly likely that Condition (a') will hold. However, it can be shown[11]

$$\lim_{n_C \to 0} \varphi = \rho$$

so that the gap in prices charged in equilibrium disappears. When n_C is exactly zero, this is the model of captive consumers and switchers studied by Narasimhan (1988) and Deneckere, Kovenock, and Lee (1992).

When n_C approaches unity, this is again the Diamond model, where all consumers must pay to search and the equilibrium outcome is monopoly pricing.

Let us finally turn our attention to the question of social welfare. Due to the unit demand structure, most effects which causes an increase in price, such as a decrease in the measure of free searchers, will cause a decrease in consumer surplus which will be exactly offset by an increase in producer surplus. Hence, the net effect on social welfare is neutral for changing most parameters of the model. There is one exception, however: an increase in search costs is, on

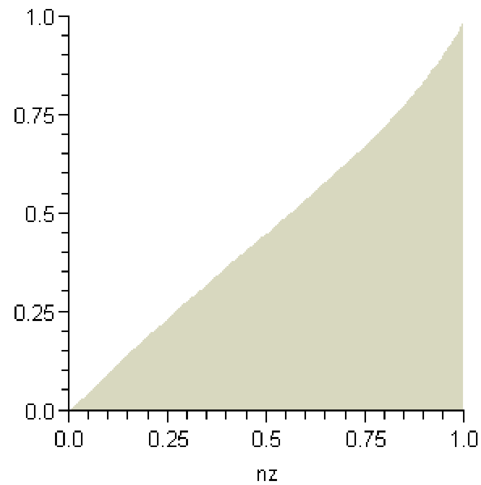
[11] Rewriting $\underline{p}$ in terms of n_Z and n_C (but not n_H):

$$\underline{p} = \frac{\rho\theta\,(2n_Z + n_C)\,(n_Z + n_C - 1)}{\rho n_C\,(1 + n_Z + n_C) + 2\theta(n_Z + n_C)\,(n_Z + n_C - 1)}$$

and taking the limit as $n_C \to 0$

$$\lim_{n_C \to 0} \frac{\rho\theta\,(2n_Z)\,(n_Z - 1)}{2\theta(n_Z)\,(n_Z - 1)} = \rho$$

Figure 3: RHS of Condition (a') as $n_C \to 0$



net, detrimental to social welfare. An increase in search costs drives up prices, which is bad for consumers and good for producers in equal measure. However, because this search cost is, with positive probability, actually incurred, there is also a direct negative effect on consumer surplus net of search costs, which is not offset by an increase in producer surplus. Recall that expected search cost is given by:

$$\begin{aligned} E(s) &= s(1 - F(\rho)) \\ &= s \cdot \frac{[\rho\,(n_Z - 1) + \theta n_H]}{\rho\,(2n_Z + n_C)} \end{aligned}$$

which is positive whenever $F(\rho) < 1$, which in turn happens whenever condition (a) holds. In this manner, search costs bring about a deadweight loss in equilibrium. In the bulk of search cost models, costly search is never undertaken. Hence, search costs, in such models, cannot have a direct effect on social welfare. This model, in contrast, brings about actual costly search in equilibrium; this costly search is detrimental to social welfare.

5 Conclusion

This paper features a model with three classes of consumer search: free searchers, moderately costly searchers, and very costly searchers. The addition of the third type of consumer allows for results which are strikingly different from those currently available in the literature. This simple (and realistic) change admits an equilibrium where consumers engage in costly search with positive probability. Additionally, the equilibrium price distribution generated by this model replicates the real-world phenomenon of sales, as noted by Varian, that prices are rarely only slightly lower than "full price." Indeed, this model generates an equilibrium where low and high, but never moderate, prices are charged. Increases (decreases) in price lead to decreases (increases) in consumer surplus which are exactly offset by increases (decreases) in producer surplus. The exception to this is in the low-and-high price equilibrium, where some costly searchers actually search; here, total surplus is decreasing in s. It would be desirable to reconfigure this model with a more general demand function; however, useful insights, namely the existence of novel equilibria have been obtained in this model. It is my belief that the general existence of the low-and-high price equilibrium and monopoly price equilibrium will continue to hold with a more general demand structure.

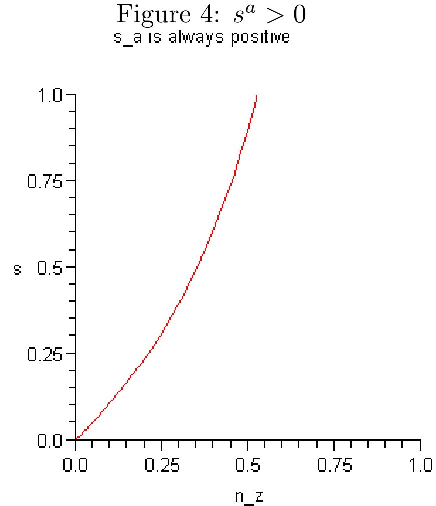
6 Appendix

We assumed in the text that $s^\star < s^a$ for some parameter values. We shall now show that this is, in fact, the case. First, note that $s^a > 0$ always. Recall s^a :

$$s^a = \frac{\theta}{n_H} \left\{ \ln \left(\frac{1-n_z}{1+n_z} \right) \left(\frac{1}{2n_z} \right) + \frac{1}{1-n_z} \right\}$$

which will be positive if and only if the term inside the brackets is positive. We can see from Figure (4) that this is always true.Next, we will show that $s^\star \geq 0$,

Figure 4: $s^a > 0$



which we will again demonstrate graphically. See Figure (5)As is clear from the above figure, as long as n_H is not very close to unity, then it is the case that $s^\star$ is positive. In fact,

$$\lim_{n_H \to 1} s^\star = 0$$

Of course, we know that as the number of high cost searchers approaches unity, this model collapses to the Diamond result of monopoly pricing. For any $n_H < 1$, it will be the case that $s^\star > 0$.
We will now present parameter values such that $s^\star < s^a$. Recall s^a which is given by equation (2) and $s^\star$ which is given by equation (4). As is shown above, both s^a and $s^\star$ are positive, and so

$$s^a > s^\star \Leftrightarrow \frac{s^a}{s^\star} > 1.$$

We provide numerical values for which this relationship holds. For a given measure of free searchers, this condition is met only if the number of very costly searchers n_H is sufficiently large. Particular parameter values for this relationship are presented in Table (1). The numbers in the matrix represent the smallest value of n_H such that $s^a > s^\star$.

Table 1: Critical Value of n_H s.t. $s^a > s^\star$

	$n_Z = 0.1$	$n_Z = 0.2$	$n_Z = 0.25$	$n_Z = 0.3$	$n_Z = 0.4$
$\theta = 1$	0.575	0.56	0.6	0.64	–
$\theta = 2$	0.56	0.48	0.47	0.5	0.6
$\theta = 3$	0.56	0.48	0.45	0.46	0.56
$\theta = 5$	0.56	0.48	0.44	0.42	0.52
$\theta = 10$	0.56	0.48	0.44	0.41	0.49

Proof of Lemma 1. At $s = s^\star$,

$$\lim_{x \to \theta} [g(x, s^\star) - x] = 0$$

Now consider some $s = s^\star + h$, $h > 0$. See that

$$g(x, s) - x = g(x, s^\star) - x - h$$

and

$$\begin{aligned} \lim_{x \to \theta} g(x, s) - x &= \lim_{x \to \theta} g(x, s^\star) - x - h \\ &= 0 - h \\ &= -h < 0 \end{aligned}$$

Thus,

$$\lim_{x \to 0} g(x, s) - x = \infty$$

and

$$\lim_{x \to \theta} g(x, s) - x < 0$$

Further, $g(x,s)$ is composed of functions which are continuous $\forall x>0$ whenever $n_Z \neq 0$. Thus, by the intermediate value theorem, when $s>s^{\star}$ there must exist some $x \in (0,\theta)$ such that $g(x,s)-x=0$ or equivalently, that $g(x,s)=x$.
■

Price Distribution where ρ is the upper bound

As noted in the text, if $s > s^a$ where s^a is given by equation (2), the equilibrium price distribution becomes "Stahl-like" and the upper bound of the support is given by ρ. We present this case here.

Claim 3 *If $s>s^a$, then the equilibrium price distribution is given by*

$$F(p)=1-\frac{1}{n_Z}\left(\frac{n_H}{2}+\frac{n_C}{2}\right)\cdot\rho \tag{18}$$

and bounds b and ρ given by

$$b = \frac{\rho\,(1-n_Z)}{1+n_Z} \tag{19}$$

$$\rho = \frac{2n_Z s}{2n_Z+\ln\left(\frac{1-n_Z}{1+n_Z}\right)\cdot(1-n_Z)} \tag{20}$$

Proof. If ρ is the upper bound of the distribution F, then if a firm charges $p=\rho$, it will sell to its very- and moderately-costly searching segments, but will be undercut with full probability, and so will never sell to the free searchers. Expected profit is given by

$$E\pi(\rho)=\left(\frac{n_H}{2}+\frac{n_C}{2}\right)\rho$$

Equating this expected profit with the expected profit from charging $p<\rho$ given by equation (8) yields the price distribution:

$$F(p)=1-\frac{\rho}{2n_Z}\left(n_H+n_C\right) \tag{21}$$

which has density function

$$f(p)=\frac{\rho\,(n_H+n_C)}{2n_Z p^2}$$

which is clearly positive. Setting equation (18) equal to zero and solving for p yields the lower bound of the support of F, denoted by b :

$$b=\frac{\rho(1-n_Z)}{1+n_Z}$$

where $b>0$ as $n_Z \in (0,1)$. Recall that ρ is given by the unique root to $\rho-E(p)-s$ where expected price is

$$\begin{aligned} E(p) &= \int_b^{\rho} pf(p)dp=\left[\frac{\rho\,(n_H+n_C)}{2n_Z}\right]\cdot\ln\left(\frac{1+n_Z}{1-n_Z}\right) \\ \Rightarrow &\quad \rho=\frac{2n_Z s}{2n_Z+\ln\left(\frac{1-n_Z}{1+n_Z}\right)\cdot(1-n_Z)} \end{aligned}$$

What remains to be shown is that such an equilibrium cannot hold if $s < s^a$ is satisfied. Assume to the contrary that $s < s^a$. If ρ is the upper bound of the distribution of F, it must be the case that $E\pi(\rho) > E\pi(\theta)$. This requires:

$$\begin{aligned} n_H\theta &< \{n_H + n_C + [1 - F(\rho)] \cdot (1 + n_Z - n_H)\} \cdot \rho \\ \frac{n_H\theta}{\rho} &< n_H + n_C + [1 - F(\rho)] \cdot (1 + n_Z - n_H) \end{aligned}$$

and because prices above ρ are never charged, $F(\rho) = 1$; the above inequality simplifies to

$$\begin{aligned} \frac{n_H\theta}{\rho} &< n_H + n_C \\ \frac{\theta}{\rho} &< 1 + \frac{n_C}{n_H} \end{aligned}$$

Rearranging terms, we see that

$$\rho < \frac{n_H\theta}{n_C + n_H}$$

and substituting ρ from equation (20):

$$\frac{2n_Z s}{2n_Z + \ln\left(\frac{1-n_Z}{1+n_Z}\right) \cdot (1 - n_Z)} < \frac{n_H\theta}{(1-\gamma)\, m + n_H}$$

and algebraic manipulation will yield the following condition:

$$s > \left[\ln\left(\frac{1 - n_Z}{1 + n_Z}\right)\frac{n_H}{2n_Z} + \frac{n_H}{1 + n_Z}\right]\theta = s^a$$

which contradicts condition $s < s^a$. Therefore, the equilibrium described by equation (18) cannot hold. ■

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Figure 5: $s^{\star} > 0$