Efficiency or Competition? A Structural Analysis of Canada's AWS Auction and the Set-Aside Provision*

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Abstract

In 2008 Industry Canada auctioned 105MHz of spectrum to a group of bidders that included incumbents and potential new entrants into the Canadian mobile phone market, raising $4.25 billion. In an effort to promote new entry, 40MHz of spectrum was set-aside for new entrants. We adapt the methodology of Bajari and Fox (2009) to the Canadian auction setting in an effort to estimate the implicit cost (in terms of lower auction efficiency) of this policy. Our results indicate that revenue would have been approximately 10% higher without the set-aside.

1 Introduction

Spectrum auctions have become an attractive tool for generating revenue for national governments. While the United States continues to operate one of the largest and most profitable spectrum auction, numerous countries have followed their lead over the past decade. Austria, Germany and India recently concluded spectrum auctions while Denmark and Mexico have spectrum auctions planned in the near future. Given the importance of cellular phones in developing countries supplying valuable social overhead capital, the provision of spectrum in Latin America (Hazlett and Muñoz, 2009) and Africa (Waverman, Meschi, and Fuss, 2005) may play a key role in promoting economic growth in the near future. Jack and Suri (2011) document the rapid growth of mobile money accounts (M-PESA) set up in Kenya through telecommunications providers and find an average of well over 100,000 daily transactions.

The recent experience of India highlights the importance of developing transparent methods for allocating spectrum. India's 3G auction concluded in May 2010 and generated $14.6 billion in revenue. This amount far exceeded expectations, which were in part based on estimates from its

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[*] Kyle Hyndman was the co-author of the report for Industry Canada which examined all aspects of the AWS Auction studied here, including bidder behaviour. We would like to thanks Sam Dinkin for his valuable comments on this paper.

allocation of 2G spectrum in 2008. Indeed, a recent report suggests that the allocation mechanism — essentially first-come first-served — for the 2G licenses may have cost the Indian government $40 billion in lost revenue and has led to a political scandal.[1] We feel safe, therefore, in our assertion that the proper design and implementation of these auctions is of paramount importance for both economic and political reasons.

At the same time, governments might have an interest in promoting downstream competition by limiting the amount of spectrum in the hands of any single firm, or otherwise trying to promote entry. This is particularly germane to the case of Canada, where the government has noted:

> [t]he smaller number of mobile providers in Canada – and the fact that all three national wireless service providers are also owned by large telecommunications service providers that also provide wireline services – may mean that there is less competition in the Canadian wireless market than in the U.S. market, which consequently has resulted in higher prices, less innovation, lower uptake and lower rates of usage. (Industry Canada, 2007, p. 3)

In an effort to promote competition, Canada's 2008 Auction for Advanced Wireless Spectrum (AWS) included a set-aside provision, which held that 40MHz of this newly available spectrum could be bid on only by new entrants.[2] They also imposed rules limiting the ability of the eventual winners of this set-aside spectrum from later re-selling it to the big 3 incumbents for a specific period of time.

While it may be hypothesized that the losses from the designated prevention of incumbent telecommunications bids in these auctions is likely to be high, Ayres and Cramton (1996) show that set-aside auctions can have several competing effects. First, set-aside auctions can enhance intragroup competition amongst incumbent firms. The incumbent firms will bid higher on those licenses for which they are eligible to win to increase the probability of winning. Second, if the proportion of the total available spectrum that are designated as set-aside is large, this can result in higher overall revenues since firms prevented from bidding in the set-aside will again bid larger amounts to increase the probability of winning in the remaining markets. If the set-aside auctions are conducted efficiently then it is possible that the revenues from the auction may be higher or at a minimum, the losses should not be too large.

Our goal in this paper is to assess the impact that the set-aside blocks in the recent Canadian spectrum auction had on the efficiency of the overall auction. While Ayres and Cramton (1996) discuss the values of the set-aside as an important policy tool for conducting efficient spectrum

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[1] See the articles, "Telecom Scandal Plunges India Into Political Crisis," *The New York Times*, December 14, 2010, A1. and "Bids Total $11 Billion for Wireless Spectrum in India," *The New York Times*, May 20, 2010, B13.

[2] See the recent article "Wireless upstarts score on new subscribers," *The Globe and Mail*, December 18, 2010 suggesting that new telecommunications firms are projected to obtain as many as one-third of new subscribers signing up for cellphone plans in the current fiscal quarter.

auctions, very little empirical (structural) work exists examining if set-asides actually yield the types of gains one would expect if they were deployed consistently. Specifically, the set-aside blocks can benefit both the government (through increased revenue) and consumers (through lower calling plan prices). A recent exception is Cramton, Ingraham, and Singer (2008) who look at price inflation in the set-aside FCC Auction 35. An incumbent, specifically prohibited from bidding in this auction, participated through a front company which not only enabled it to win more licenses at lower prices, but also increased the prices that other non-incumbents won. They find that prices were substantially inflated given the participation of the incumbent telecommunication firm, thus eliminating one of the intended effects of the set-aside.

Our most plausible estimates show a maximal possible efficiency gain of about 30% (or $1.28 billion). However, this scenario presumes that a single bidder wins all of the spectrum that was sold in the auction — something that is most unlikely under any realistic scenario. A more likely outcome, in the absence of a set-aside, we believe is for the three incumbents to have won all of the spectrum. Under this scenario, we calculate an efficiency gain on the order to 6.7 to 11.1%. The upper bound of this range translates into approximately $471 million. While we cannot estimate the benefit to consumers from any increased competition that the auction was designed to promote, it is plausible that the gains to consumers may be more than the $471 million in lost revenue.[3] As of the end of 2010, there were at least 3 new wireless providers operating in various cities across Canada. Both Wind Mobile (Globalive) and Mobilicity (DAVE) operate in Toronto, Ottawa, Edmonton, Calgary and Vancouver, while Public Mobile has operations in Toronto and Montreal. A fourth, Vidéotron (Quebecor) began providing services in Québec on its own network, where it had previously offered service using the Rogers network. Shaw was expected to launch its network in Western Canada in 2011, but recently announced that the launch would be delayed until 2012.[4] Generally speaking, Wind Mobile, Mobilicity and Public Mobile advertise themselves as lower cost, more transparent alternatives to the three nationwide incumbent operators. Recognizing the stakes involved, incumbents and entrants alike have challenged a government ruling which allowed Wind Mobile (Globalive) to continue to operate despite an earlier ruling by the telecommunications regulator that it did not meet the Canadian ownership requirements for telecom companies.[5] This comes shortly after Wind Mobile announced that it has exceeded 250,000 wireless customers.

While a burgeoning literature on the structural estimation of auctions exists (see the special issues on the "Econometrics of Auctions" in the *Journal of Applied Econometrics* (Dubois, Ivaldi, and Magnac (2008)) for a bevy of examples and references to the literature) very little structural empirical work on spectrum auctions exists. An exception is the elegant study of Bajari and Fox

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[3] For example, in 2006, it was estimated that there were approximately 18.6 million cell phones in use in Canada. If prices drop by an average of $1 per cell phone per month, the effect of the loss in auction efficiency will be offset in only two years.

[4] See the article, "Shaw network delayed, but primed for 4G push," *The Globe and Mail*, 13 January 2011.

[5] See the article, "Canada court overturns government ruling on Globalive," *Reuters*, 4 February 2011.

(2009) who investigated the C block of the US spectrum auction conducted between 1995 and 1996. Their findings suggest that geographic complementarities across licenses were a significant component of a telecommunications firm's decision to bid. Moreover, the results of their model imply that geographic complementarities make up approximately 41% of a package's value to any given firm. To assess efficiency of the C block spectrum auction Bajari and Fox (2009)'s counterfactual analysis found that the partition of licenses across the C block did not maximize potential benefits arising from geographic complementarities. Instead, an auction consisting of 4 large regions would have roughly doubled the value arising from complementarities relative to the actual C block setup. Moreover, partitioning the US into these four large regions would have significantly raised the proportion of the US population that was won by high-value bidders.

However, this study differentiates itself with Bajari and Fox (2009) in several respects. First, this marks the first academic study of the Canadian spectrum auctions. Additionally, we consider simultaneously numerous blocks of the Canadian spectrum auction which allows us to consider both vertical and horizontal differentiation of the spectrum (Bajari and Fox only consider horizontal differentiation). We use the amount of total spectrum owned in a given area to construct appropriate measures of geographic complementarities and initial eligibility across our auctions. In today's telecommunications environment having sufficient spectrum is paramount since firms typically offer both phone and data plans. Thus, it is imperative to recognize the role that the level of spectrum plays when considering geographic complementarities. While Bajari and Fox (2009) note that geographic complementarities capture more than just the desire of customers to make calls when traveling[6] these complementarities are even more potent when one considers that enough spectrum must exist for customers to surf the web, send photos and text messages, and check their e-mail. Our modified measures will be seen to capture these additional components of geographic complementarities.

Second, while we employ the structural matching estimator of Fox (2009) (also used in Bajari and Fox) we include alternative measures of competition in the estimator which allows us to consider other factors for telecommunications' decisions to purchase spectrum outside of geographic complementarities. While we include measures of competition across the packages of licenses constructed by the firms, we also introduce a new measure of competition that is based on overlap between the packages instead of the common concentration measures. These overlap measures are commonly employed in the healthcare literature as a means to measure competition for different healthcare services amongst hospitals. Here we use these insights to construct overlap of spectrum between packages. Lastly, we use the same econometric methodology as Bajari and Fox (2009) but we also implement a smoothed maximum score estimator as a basis for comparison. To our knowledge we are unaware of empirical studies that deploy both estimators and as such this represents a useful comparison.

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[6] Such as cost of synergy and marketing.

Our baseline results suggest that our spectrum modified measures of initial eligibility and geographic complementarities improves the number inequalities satisfied in the maximum score estimator by over 6 percentage points relative to the exact measure of geographic complementarities suggested by Bajari and Fox (2009). Second we find the expected signs for both measures of geographic complementarities. Furthermore, we see that geographic complementarities represent a large component of the value that telecommunications firms place on their packages of licenses for Canadian spectrum. When we include alternative measures of competition this results in a model which fits the data slightly more accurately. Moreover, as one might expect, the estimated coefficient on competition is negative, and, for one measure of competition, very often statistically significant.

Our counterfactual analysis to investigate the efficiency properties of the set-aside is dependent upon which measure of geographic complementarities we use. The non-spectrum adjusted measure yields comparable results to those for the C block auction studied by Bajari and Fox (2009) while our new measure suggests that the loss in efficiency is much smaller (approximately 10%). The inclusion of competition, while producing the appropriate sign appears to have a muted effect on the overall value bidders place on their packages, with geographic complementarities and the amount of spectrum won comprising a larger portion of a bidder's valuation for a given package. Overall, the results here suggest that telecommunications firms not only value a large geographic footprint, but that an adequate level of spectrum is required when constructing the optimal package.

The remainder of the paper is structured as follows. Section 2 describes the Canadian AWS auction in detail. Section 3 discusses the pairwise matching estimator which will be used to construct our structural profit function. Section 4 details our construction of geographic complementarities as well as two different measures of competition. Section 5 presents our econometric results and several counterfactual exercises designed to gauge the effectiveness of the AWS auction. Section 6 concludes.

2 Canada's AWS Auction

Industry Canada's auction of Advanced Wireless Spectrum (AWS) took place between May 27, 2008 and July 21, 2008. In total 90MHz of AWS spectrum in the 2GHz range, as well as 10MHz of the PCS Expansion Band and the 5MHz band in the 1670-1675 MHz range were up for auction. Although 27 bidders submitted qualifying applications, only 21 bidders actively participated in the auction.

One of the important features of this auction was that 40MHz of AWS spectrum was set aside exclusively for those firms labeled as new entrants, where new entrants were defined as those bidders who had less than 10% of the national market by revenue. In effect, this rule excluded the three large national wireless operators (Bell, Telus and Rogers) who, according to Industry Canada, control 94% of the market, while at the same time allowed two smaller regional incumbents, SaskTel and MTS, to bid in the set-aside (Industry Canada, 2007, p. 3). The remaining 50MHz of AWS spectrum and 15MHz of additional spectrum was open to both the three large incumbents as well as the new entrants.

The auction lasted 331 rounds, spread out over 39 bidding days. In total, 282 of the 292 licenses (including all of the AWS spectrum) up for auction were sold, with 15 different bidders winning licenses. The auction generated approximately \$4.25 billion in sales of licenses and withdrawal penalties — an amount that nearly tripled initial revenue expectations of \$1.5 billion.[7]

2.1 Details on Spectrum Blocks

The spectrum up for auction was divided into 8 blocks of various sizes. Blocks A – F were the AWS spectrum blocks, while block G was for the PCS Expansion Band and block I was for the 1670-1675MHz spectrum. Table 1 outlines the important details of the auction. Each block represented a range of the spectrum, with the size represented in MHz. Each block was then partitioned into individual licenses for a given geographical area. For this auction, some spectrum blocks were partitioned into Tier 2 licenses, while other blocks were partitioned into Tier 3 licenses. Tier 2 licenses partition Canada into 14 distinct economic regions, while Tier 3 licenses partition Canada into 59 smaller areas — basically metropolitan areas. As can be seen, Blocks B, C, G and I were partitioned into Tier 2 licenses and all other blocks were partitioned into Tier 3 licenses.[8]

Table 1: Auction Details

Block	A	B	C	D	E	F	G	I
MHz	20	20	10	10	10	20	10	5
Tier	3	2	2	3	3	3	2	2
Spectrum	Open	S/A	S/A	S/A	Open	Open	Open	Open

"Open" means that all bidders were eligible to bid on the spectrum, while "S/A" means that this spectrum was set-aside for new entrants (with 3 incumbents forbidden from bidding).

To get a sense of the notation that we will use, the set of licenses in the B block are denoted, $\{201b, 202b, \ldots, 214b\}$ and similarly for the C, G and I blocks. Similarly, the set of licenses in the A block are denoted, $\{301a, 302a, \ldots, 359a\}$ and similarly for the D, E and F blocks. Therefore, it should be understood that the first digit represents the tier structure, the second and third digits represent the geographical area, while the letter denotes the frequency block. Note that the tier 3 licenses can be viewed as a refinement of the partition of tier 2 licenses. Thus, for example, license

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[7] See the article, "Options abound as auction kicks off," Financial Post, 27 May 2008, in which it was reported that "[t]he auction is expected to raise about \$1.5 billion for the federal government."

[8] The fact that different blocks had different tier structures, or partitions, creates some issues in the construction of our variables.

203c and the set of licenses 305d, 306d and 307d are equivalent in the sense that they would each give the winner 10MHz of spectrum covering the province of New Brunswick.[9]

2.2 Bidding Behaviour: Signaling

Bajari and Fox (2009) propose an estimation procedure for bidder valuations based on a particular notion of pairwise stability. This condition implies that two bidders cannot exchange licenses in a way that increases total surplus. Bajari and Fox (2009) demonstrate that their condition is satisfied in a number of theoretical models of simultaneous ascending auctions, including some models of bidder intimidation and demand reduction.

Given the transparency of spectrum auctions in practice (*e.g.,* all bids, including the identity of the bidder, are made public at the conclusion of each round), bidders may have many signaling opportunities that could be used to coordinate a mutual demand reduction or also to signal strength. Two ways in which a bidder may attempt to signal are via so-called jump bids and what we call *tit-for-tat* bids. In order to motivate our subsequent use of Bajari and Fox's notion of pairwise stability, we will briefly discuss each of these in turn.

2.2.1 Jump Bids

Bidding in the AWS auction was "yes/no". That is, for each license, the auction software listed the the current price p of the standing high bidder and a price $p' > p$. If bidders are willing to pay p' for the license, then they select "yes" for that particular license. The bid, p, of the standing high bidder is automatically carried forward to the next round. However, the standing high bidder may also choose to bid p' if they so desire. Such a bid is called a jump bid. By placing a jump bid, a bidder may be signaling to others that it has a high valuation for that license in an attempt to discourage others from competing for it.

In this auction, a total of 107 jump bids were placed — 58 of which were unique.[10] However, it is difficult to determine conclusively that these were done with the intention of signaling. For example, for 56 of the 107 jump bids, there was at least one other bid placed on the license. Therefore, the jump bid may have been placed with an eye towards coming out on top of a tie-break.[11] Furthermore, placing jump bids did not ensure that a bidder would eventually win the particular license — in only 20 instances did the jump bidder eventually win and in only two cases was a jump bid the final bid placed. Indeed, it seems that the main use for jump bids was to

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[9] That is not to say that the price of license 203c should equal the sum of prices of 305d, 306d and 307d. In particular, because of the exposure problem, a bidder who wants to win a license covering New Brunswick may be willing to pay less for the tier 3 licenses because of the inherent risk in not winning the complete package.

[10] There were instances in which the same bidder placed multiple jump bids on the same license in different periods.

[11] For example, near the end of the auction, prices may be coming close to one's valuation. Therefore, placing a jump bid (and winning a tie-break) may allow the bidder to win the license and save one bid increment, which, in this auction was at least 4%.

prevent the auction from closing prematurely before bidders had achieved their desired footprint. For example, in two rounds a jump bid was the only bid placed and in the final 100 rounds of the auction, jump bids represented approximately 10% of the total number of bids.

2.2.2 Tit-For-Tat Bids

Consider now tit-for-tat bidding, which we define as follows. Let $S(a, N)$ denote the set of licenses for which bidder a is the standing high bidder in round N. Let $S(a, N, T)$ denote the set of licenses for which bidder a either bid on, or was the standing high bidder on, in rounds $N-t, N-t+1, \ldots, N$. Then suppose that bidder b outbids bidder a in round N on some set $O(b)$, which is a subset of $S(a, N)$. We will say that bidder a places a tit-for-tat bid against bidder b if, in round $N+1$, bidder a bids on licenses in the set $S(b, N+1) \setminus S(a, N, t)$ (i.e., bidder a bids on those licenses that bidder b is the standing high bidder on at the beginning of round $N+1$, and which bidder a has not bid on, nor was the standing high bidder, in the previous t periods).

If we take $t = 10$, then there are approximately 720 instances of tit-for-tat bidding, while if we take $t = N$ (i.e., so that the bidder who places a tit-for-tat bid has never bid on that particular license until round N), then the number of instances decreases to 336. Even for these 336 tit-for-tat bids, it is difficult to say that they were placed with an eye towards signaling. For example, 111 of the bids in question involved bids between the incumbents, primarily on licenses in the A, E and F blocks, which suggests that price arbitrage may have been the primary motivation for such bids.

On the other hand, there are some instances in which apparent tit-for-tat bids did appear to represent attempts to signal to other bidders one's intentions. We highlight three such examples. First, in round 19, Bragg outbid Shaw on license 209b. In the next round, Shaw placed a bid on license 202b, which, it is arguable, Shaw had no interest in obtaining because it did not appear to be consistent with Shaw's other bids.[12] Although not picked up by our strict definition of a tit-for-tat bid, the exact same scenario played out again in rounds 71 and 72. In this case, it would appear that Shaw's attempt to signal its desire for license 209b to Bragg had very little effect, except to increase the price that Bragg paid for licenses 202b and 209b, since it eventually won both licenses.

Second, in round 71, Globalive outbid Quebecor on a number of licenses in Quebec. In the following round, Quebecor outbid Globalive on licenses in Atlantic and Western Canada (i.e., 201c, 203c, 302d, 303d, 304d, 339d and 340d), all of which Quebecor had never bid on before that round (and likely had little interest in actually winning, since Quebecor is a company whose primary interest lies in the province of Québec). Interestingly, after round 72, while Globalive continued to seek out a foothold in Quebec, it only placed bids for non-set-aside licenses.

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[12] Indeed, see the article, "Winning air waves with poker-like skills; Corporate strategy. How Shaw Communications broke into the mobile phone business without losing its shirt," *The Globe and Mail*, July 26, 2008, in which Shaw discusses its bidding strategy.

Finally, to give an example where incumbents may have engaged in tit-for-tat bidding with an attempt to signal, we note that in round 66 Bell outbid licenses held by Rogers. In the following round, Rogers placed bids on licenses 318f, 319f and 320f. This occurred under the following conditions (i) Rogers had never previously bid on these licenses, (ii) that Rogers was the current standing high bidder on the equivalent licenses in the A block and (iii) that the current prices were substantially higher in the F block than in the A block.

Thus, while it is difficult to conclusively determine the exact intensity with which bidders engaged in signaling, either through jump bids, tit-for-tat bidding or some other means, it does appear that some signaling did occur during the auction. For this reason, we will use Bajari and Fox's notion of pairwise stability, which, in some instances, is robust to such behaviour.

2.3 Complementarities

We now briefly discuss some suggestive evidence that complementarities can be expected to play a role in our subsequent data analysis. For example, the two winningest bidders (in terms of dollars) both won licenses covering all of Canada, while the third and fourth winningest bidders appeared to have tried, though failed, to obtain a national footprint. For these bidders we expect there to be strong geographic complementarities. On the other hand, while geographic complementarities are likely present for the other bidders, their magnitude may be limited for other reasons. A number of the bidders given new entrant status were cable TV or regional telecommunications companies. These bidders already have geographic footprints in their respective markets.[13] For these bidders, their primary motivation may have been to gain spectrum in their existing geographic markets in order to be able to improve service or to provide a more comprehensive service (*e.g.,* cable, internet, telephone and wireless) to their existing pool of customers. To the extent that this is true, we would expect geographic complementarities to be somewhat mitigated.

The geographic footprints of four winners are given in Figure 1. The darker the shade of blue, the more frequency that was won in a given area. Those areas in white indicate that the bidder did not win any spectrum in that region. As can be seen, with some exceptions, Shaw, Quebecor and Bragg did appear focused primarily on winning spectrum in their existing markets.

2.4 Winners

Figure 2 shows a scatter plot of bidders' winning eligibility (in logs) as a function of their initial eligibility (in logs), as well as the regression line that best fits the data. As can be seen, it is generally true that bidders with more initial eligibility won more licenses. Moreover, it is also true

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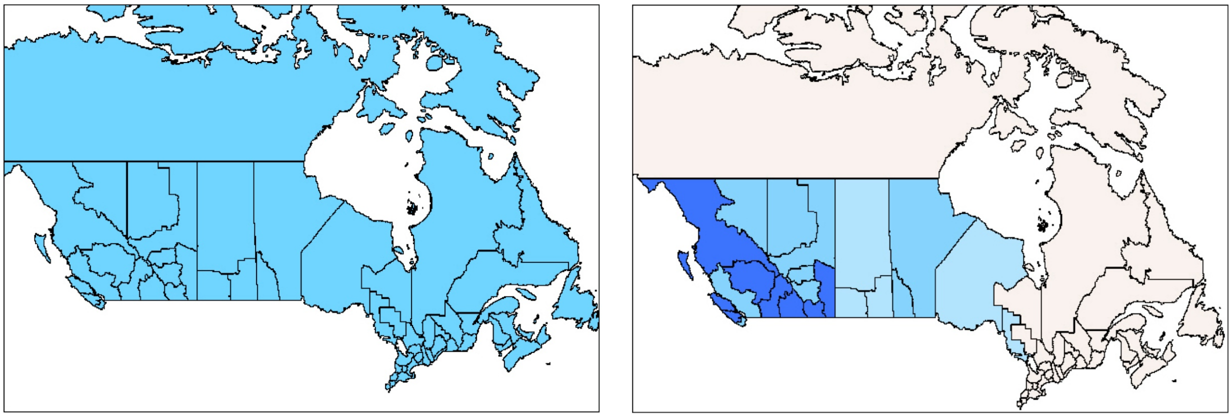
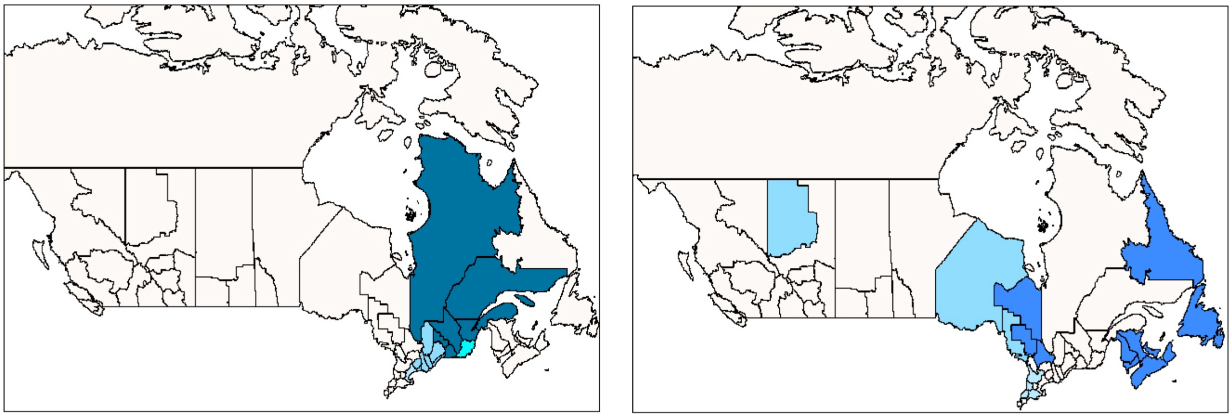
[13] For example, Shaw, Quebecor and Bragg are cable TV and internet service providers whose existing geographic footprints are concentrated in Western Canada and Northern Ontario (Shaw), Québec (Quebecor) and Atlantic Canada (Bragg), respectively. Similar, SaskTel and MTS are telephone and internet service providers for the provinces of Saskatchewan and Manitoba, respectively.

Figure 1: Geographic Footprints of Select Winning Bidders



(a) Rogers Communications

(b) Shaw Communications

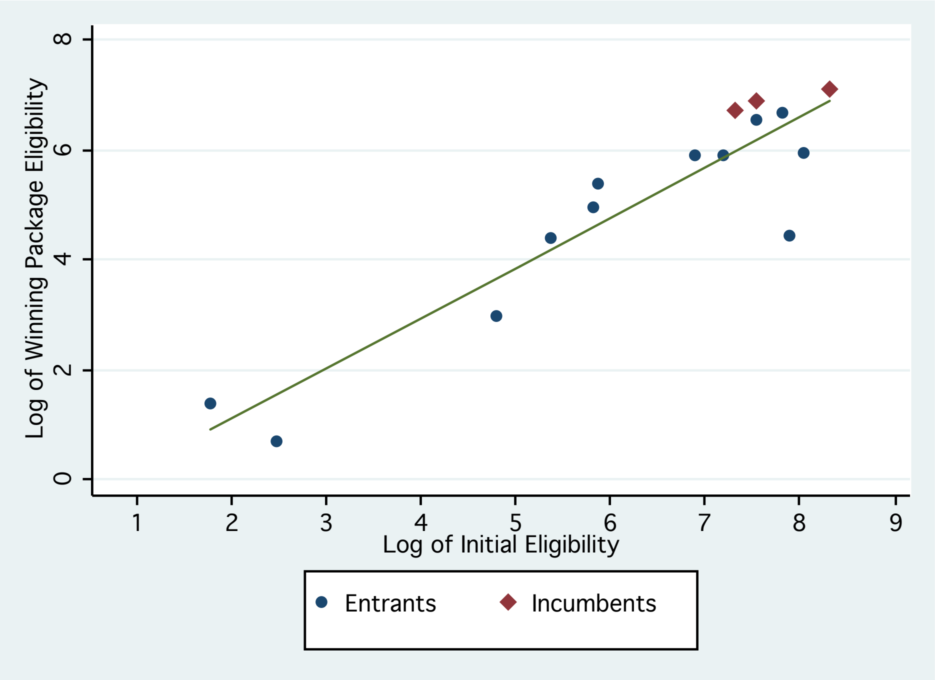


(c) Québecor

(d) Bragg Communications

that the three incumbent bidders won the most spectrum (in terms of eligibility points), though they were closely followed by two entrants, with the remaining bidders winning moderately to substantially less spectrum.[14]

Figure 2: Winning Package Eligibility vs. Initial Eligibility



Like Bajari and Fox (2009), we will use initial eligibility (suitably normalized) to capture bidder characteristics. Even more so than in the FCC C Block auction that they analyze, here initial eligibility should be a very good proxy for the financial strength of the bidders since the required deposits were relatively large. For example, Rogers made an initial deposit of approximately \$534 million and won licenses totaling approximately \$999 million. Additionally, within 10 days after the close of the auction, bidders had to pay 20% of their winning bid total, and the remaining 80% within 30 days. The penalty for not making the appropriate payments on time was the forfeiture of all licenses plus additional penalties.

3 The Matching Estimator of Fox (2009)

Let L denote the number of licenses for sale and $a = 1, \dots, N$ the number of bidders. Fox (2009) proposes a maximum score (or maximum rank) estimator (Manski (1975), Han (1987)) which is designed to minimize the number of times two different bidders would want to trade a single license

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[14] In particular, the 6th winningest bidder won only 55% of the spectrum won by the 5th winningest bidder and only 30% of the spectrum won by the winningest bidder.

from the package of licenses with each other. If the bidders had truly constructed their optimal packages then no trades would be agreed upon. The objective function for this problem is

$$Q(\beta) = \frac{2}{L(L-1)} \sum_{i=1}^{L-1} \sum_{j=i+1}^{L} 1\left\{a(i) \neq a(j)\right\} \cdot 1\left\{\mathcal{I}_{ij}(\beta)\right\}, \tag{1}$$

where $a(k)$ denotes the bidder who won territory k and $\mathcal{I}_{ij}(\beta)$ is an inequality defined over the matching between license i and license j. This function solves for β by considering all combinations of two licenses that are won by different bidders. The matching inequalities can be set up as

$$\begin{aligned}\mathcal{I}_{ij}(\beta) = m(a(i), J_{a(i)}; \beta) + m(a(j), J_{a(j)}; \beta) \geq \\ m(a(i), (J_{a(i)} \backslash \{i\}) \cup \{j\}; \beta) + m(a(i), (J_{a(j)} \backslash \{j\}) \cup \{i\}; \beta).\end{aligned} \tag{2}$$

The notation $(J_{a(i)} \backslash \{i\}) \cup \{j\}$ implies that the package of licenses obtained by the winning bidder in territory i has one license replaced by one of the winning licenses for one of bidder j's licenses. Once $m(a(i), J_{a(i)}; \beta)$ is specified, $Q(\beta)$ can be optimized using non-gradient optimization algorithms such as Nelder-Mead or Differential Evolution.

This estimator is econometrically useful under the current setting where pairwise stability is assumed to hold. In essence pairwise stability generates a set of rank order conditions which generate an equilibrium sorting pattern in the data which can then be exploited to obtain coefficient estimates for the structural profit function. More generally, these pairwise stability conditions also allow for the inclusion of license specific fixed effects which are removed in the differencing procedure that occurs when constructing the inequalities above. This is identical to the process of removing the cross section dimension fixed effects in a linear panel data model. For the Canadian AWS auctions we believe that pairwise stability is satisfied.[15]

Note however, that not all pairwise trades of licenses would be allowed by the auction rules. In particular, the B, C and D blocks were available only to designated entrants, while the A, E, F, G and I blocks were open to both entrants and incumbents. Therefore, it would not be possible for an entrant winning a B-block license, for example, to trade it for a license held by an incumbent.

To obtain confidence intervals for our estimates of β one could resort to asymptotic distributional results. However, the limiting distribution of the maximum score estimator is quite difficult to approximate in practice. As such two main alternatives exist. First, one could deploy a subsampling mechanism, say Politis and Romano (1994), or a smoothed maximum score estimation routine could be invoked which allows for valid bootstrap confidence intervals to be constructed. We outline each of these approaches here.

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[15] The fact that no licenses were resold immediately after the conclusion of the auctions, and that no mergers between firms has taken place in the several years since the auctions were held provide anecdotal evidence that pairwise stability is a reasonable assumption for the AWS auctions.

The subsampling algorithm of Politis and Romano (1994) (advocated by Fox (2009)) is simple to implement. The only thorny issue to deal with is how to take the subsamples. In a standard regression setting the subsample of Politis and Romano (1994) would fix a subsample size and then draw repeated samples without replacement of that size, reestimate the model and then construct confidence intervals. However, in the current setting our observations are not *iid* draws from a cross sectional distribution. Rather, these observations are linked via the exchange of licenses across two packages. The approach for subsampling in this context is to take subsamples of the *packages* as opposed to the actual matches. For example, in our setting we have 15 total packages, so a subsample of 6 packages would produce a total number of observations to construct the matching function equal to the total number of pairwise license exchanges across those 6 packages.[16]

To detail the construction of the subsampled confidence intervals we use the following notation. Let the total number of matches be denoted as n and the size of a given subsample be n_s. We let $\hat{\beta}_{MS}$ denote our original maximum score estimate and $\hat{\beta}_{SS}$ be our subsampled maximum score estimate. The subsampling process to construct a $1-\alpha$ confidence interval that we use here is as follows:

1. Sample without replacement from the packages.

2. Estimate β for the structural profit function using the subsampled packages.

3. Construct $TE = n_s^{1/3}(\hat{\beta}_{SS} - \hat{\beta}_{MS})$.

4. Repeat steps 1-3 B times.

5. Find the $\alpha/2$ and $1-\alpha/2$ quantiles of TE. Denote these values as $TE_{[\alpha/2]}$ and $TE_{[1-\alpha/2]}$, respectively.

6. The $1-\alpha$ confidence interval of $\hat{\beta}_{MS}$ is

$$n^{-1/3}[\hat{\beta}_{MS} - TE_{[\alpha/2]}, \hat{\beta}_{MS} - TE_{[1-\alpha/2]}]. \tag{3}$$

Alternatively, to make the objective function smooth one could follow Horowitz (1992) and convert the indicator function into a smooth, p-th differentiable function; replacing $1\{\mathcal{I}_{ij}(\beta)\}$ with $G_{ij}(\mathcal{I}_{ij}(\beta)/h)$ where $\lim_{v\to-\infty} G_{ij}(v) = 0$ and $\lim_{v\to\infty} G_{ij}(v) = 1$ and h is the bandwidth. This implies that inequalities that are not satisfied, $v < 0$, produce little weight whereas inequalities that are satisfied to a large degree, v large, provide a large amount of weight to the estimator. The simplest kernel function to use is the standard normal distribution function, $G_{ij}(v) = \int\limits_{-\infty}^{v} \phi(u)du$,

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[16] Bajari and Fox (2009) consider an alternative subsampling mechanism by sampling from the licenses and then constructing the full packages from those licenses. We do not employ this mechanism here since Fox and Bajari have 85 bidders and we only have 15, meaning that for many of our subsamples, the full sample would be recreated.

where $\phi(u)$ is the standard normal density, i.e. $\phi(u) = \left(\sqrt{2\pi}\right)^{-1} e^{-u^2/2}$. This kernel is infinitely differentiable and in this setting Horowitz (1992) has shown that the smoothed maximum score estimator has $\sqrt{n}$-convergence as opposed to $\sqrt[3]{n}$. An appealing aspect of the smoothed maximum score estimator is that for $h \to 0$ $G_{ij}(\mathcal{I}_{ij}(\beta)/h) \to 1\left\{\mathcal{I}_{ij}(\beta)\right\}$ and so our smoothed maximum score (SMS) estimates can be made to be arbitrarily close to our MS estimates. The advantage of using an SMS estimator is that one can employ the bootstrap since the limiting distribution is normal. This allows direct comparisons between the subsampled confidence intervals constructed for the MS estimator and the bootstrapped confidence intervals constructed for the SMS estimator.

A key issue with use of the SMS estimator is the selection of the smoothing parameter which dictates the behavior of this estimator. Given that no fully automatic data-driven approach exists to select the appropriate bandwidth for the SMS estimator we elect to use an *ad hoc* approach adopted from the kernel density literature. A common bandwidth mechanism for use in applied density analysis is to use a rule-of-thumb bandwidth which typically takes the form $h = c\sigma_x n^{-\kappa}$ where c is a constant depending upon the underlying true density, σ_x is the standard deviation of the data and n is the sample size. The $n^{-\kappa}$ that appears in this bandwidth is typical of kernel smoothed estimators and is commonly termed the rate. If the rate is too fast, the bandwidth converges to zero quickly, which reduces bias at the expense of introducing additional variance. If the rate is too slow, the bandwidth converges to zero slowly and bias is sacrificed to decrease the variance.

Horowitz (2002) shows that using a bandwidth with the optimal rate for the SMS estimator produces an asymptotic bias and that engaging in undersmoothing can remove this bias at no cost to the coverage of the confidence interval.[17] Horowitz (2002, Theorem 1.1) shows that for any $s \geq 2$, if $nh^{2s+1} \to \lambda$, then the SMS estimator will possess an asymptotic bias. If $\kappa > 1/(2s+1)$, so that the estimator is undersmoothed, no asymptotic bias will arise. For $s = 2$ we have the common $n^{-1/5}$ rate that is prevalent in univariate theoretical work. $\kappa = 1/3$ or $1/4$ will produce a sufficiently undersmoothed estimator to eliminate the asymptotic bias. We use $\kappa = 1/3$ in the empirical section of this paper. Our empirical bandwidth is constructed by taking the standard deviation of the values inside the kernel smoothing function evaluated at the MS estimates. We use the Silverman convention and set $c = 1.06$.

4 Construction of the Structural Profit Function

The structural profit function used by Bajari and Fox (2009) takes the following general form:

$$m_\beta(w_a, x_J) = \pm 1 \cdot e_a \cdot \left(\sum_{j \in J} pop_j\right) + \beta' complem_J \tag{4}$$

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[17] See also Hall (1992).

where $w_a = \{e_a\}$ represents the initial eligibility of bidder a (in terms of what fraction of the population the bidder is eligible to win) and $x_J = \left\{\{pop_j\}_{j=1}^{J}, complem_J\right\}$ contains information about certain characteristics of a winning package, J. The interaction between initial eligibility and $\sum_{j \in J} pop_j$ captures the observed finding that bidders with more initial eligibility won more licenses. The main measure of complementarities used by Bajari and Fox (2009) is that of *geographic* complementarities, which are given by:

$$geo_J = \sum_{i \in J} pop_i \left(\frac{\sum_{j \in J, j \neq i} \frac{pop_i pop_j}{dist_{ij}^{\delta}}}{\sum_{j \in L, j \neq i} \frac{pop_i pop_j}{dist_{ij}^{\delta}}} \right), \tag{5}$$

where pop_i is the fraction of the population that reside in the region covered by license i, $dist_{ij}$ is the distance (in kilometers) between licenses i and j and $\delta = 4$. An appealing feature of this measure of complementarities is that its form is similar to that arising from the gravity equation in international trade in addition to the fact that adding more licenses can never decrease a firm's complementarities using this measure.

We take a very similar approach to Bajari and Fox (2009); however, because of differences in the FCC C-Block Auction that they analyze and Industry Canada's AWS action that we analyze, there are a few differences, which could, in principle, matter. First, and rather trivially, eligibility in the AWS auction was measured in points, and each license had an associated number of eligibility points.[18] Our measure of initial eligibility is given by the each bidder's proportion of total initial eligibility. Second, and more importantly, rather than the single license covering each geographical area, the AWS Auction had 8 licenses available in each area. Furthermore, some blocks of licenses were Tier 3 licenses (smaller metropolitan regions), while others were Tier 2 licenses (larger economic regions).

The issue of different blocks having different tier structures arises in the computation of geographic complementarities. For example, suppose that bidder i has the package $J_i = S \cup \{211c\}$, while bidder j has the package $J_j = S \cup \{341d, 342d, 343d\}$. In this example, license 211c corresponds to the Tier 2 license giving 10MHz of spectrum to the province of Saskatchewan, while licenses 341d, 342d and 343d also provide bidder j with 10MHz of spectrum covering the province of Saskatchewan, although they do so with Tier 3 licenses. Intuitively, bidders i and j have exactly the same geographic footprint, which means that the measure of geographic complementarities should also be the same. Our calculation of geographic complementarities is done at the Tier 3 level. That is, we break down Tier 2 licenses into their constituent Tier 3 parts and then proceed with the appropriate computation.[19]

The issue of having multiple licenses per geographic region creates a number of potential issues. First, the amount of spectrum won by a single bidder in any one region should enter into the

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[18] Roughly, each eligibility point corresponds to a population of 100,000 per 5 MHz of spectrum.

[19] Of course, when we consider pairwise trades of licenses involving a Tier 2 license, the entire license is traded.

structural profit function. However, the question is whether it matters independently of geographic complementarities, or whether it works in a complementary manner. For example, one might imagine that complementarities are strengthened the more spectrum one has in a given area because it allows the provider to offer more services to customers.[20] On the other hand, it could simply be that more spectrum in a region allows the firm to provide better service in that region, independently of any complementarities.

If we assume that the amount of spectrum does interact with geographic complementarities, then it is natural to modify (5) by including (in the outer summation) the total amount of spectrum won by the bidder in each region:

$$geo_J^M = \sum_{i \in J} pop_i MHz_i \left(\frac{\sum_{j \in J, j \neq i} \frac{pop_i pop_j}{dist_{ij}^{\delta}}}{\sum_{j \in L, j \neq i} \frac{pop_i pop_j}{dist_{ij}^{\delta}}} \right). \tag{6}$$

But for the inclusion of MHz in the outer summation, everything else is the identical to (5).[21] From now on we will refer to (6) as *spectrum-weighted geographic complementarities* and (5) as *unweighted geographic complementarities.*

On the other hand, if the amount of spectrum does not interact with geographic complementarities, then it makes sense to maintain (5) but to also include a variable that captures the amount of spectrum won by a bidder. The most natural such measure would appear to be some population-weighted average across the 59 tier 3 license regions. That is,

$$spec_J = \sum_{i=1}^{59} pop_i MHz_{J,i}^2. \tag{7}$$

where $MHz_{J,i}$ is the amount of spectrum that package J contains in tier 3 region i.

Another issue that arises due to the multiplicity of licenses in each region is that the number of license holders in a region can likely be expected to effect the competitiveness of that region's market, which could also affect the valuation of a particular license, or package of licenses, for a bidder. To create a proxy for the competition within a given geographic area we turn to the common Herfindahl index. Our Herfindahl index for each of our 59 metro regions is calculated as

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[20] For example, consider two firms with identical geographic footprints, but that one firm has at least as much spectrum (and sometimes more) in every license area. Given this, the firm with more spectrum should be able to provide a higher level of service over its entire footprint, making it a more attractive service provider to potential customers. Having identical amounts of spectrum across one's service area may also lead to additional cost savings, such as marketing, because the provider need not tailor its marketing strategy according to the service level it can provide in each region.

[21] In our empirical work, we experimented with non-linear versions of (6), such as having MHz enter as a quadratic or square root. In all such cases, the fit was worse than the linear-in-MHz specification that we have adopted; nor were there any qualitative differences.

the sum of squares of each of the winners' share of available spectrum in region K, i.e.,

$$H_K = \sum_{k \in w_K} \left(\frac{MHz_k^K}{\sum_{\ell \in w_K} MHz_\ell^K} \right)^2, \tag{8}$$

where w_K represents the set of bidders who won at least one license in region K. Denote by H_K^{-j} the concentration of spectrum won in region K by bidders *other than* j. That is,

$$H_K^{-j} = H_K - \left(\frac{MHz_j^K}{\sum_{\ell \in w_K} MHz_\ell^K} \right)^2.$$

Our measure of competition is then given by:

$$comp_J^H = \sum_{i \in J} pop_i H_i^{-j}. \tag{9}$$

That is, we subtract out bidder j's contribution to the overall concentration in each region and take the weighted sum across all region in which bidder j won at least one license.

Lastly, we construct a completely different form of competition across the spectrum based on overlap with one's competitors in each region. This overlap measure, termed niche overlap is popular in the healthcare literature (Sohn, 2002). To construct our measure of niche overlap we first introduce some notation. Consider for each package $p = 1 \ldots, 15$ the total MHz purchased over the $r = 1, \ldots, 59$ regions. This is a 15×59 matrix whose entries present the total MHz awarded in a given region over the 8 blocks of the AWS auction. These elements will be denoted as m_{pr}. A simple measure of overlap between packages p and q in region r is $\min(m_{pr}, m_{qr})$. Sohn (2002) introduces the weighted overlap measure between packages p and q as

$$C_{pq} = \frac{\sum_{r=1}^{59} w_{pr} \min(m_{pr}, m_{qr})}{\sum_{r=1}^{59} w_{pr} m_{pr}}. \tag{10}$$

The weights, w_{pr} indicate the importance of each region to the package. The numerator provides a total overlap between two packages. If two packages did not possess spectrum in any region simultaneously the overlap would be zero whereas if the q^{th} package had more spectrum in every region the overlap would be one. Our empirical work assumes that $w_{pr} = pop_r$.[22] That is, we weight each region by the fraction of the total Canadian population that the region represents.

An interesting feature of this measure is that it is asymmetric, which our Herfindahl indices are not. These overlap measures can be interpreted as the proportion of MHz that a telecommunications

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[22] Qualitatively similar results are obtained if, instead, $w_{pr} = m_{pr} \cdot pop_r$. Such results are available upon request.

firms has in common with a competing telecommunications firm. This measure of competition may be more interesting than a concentration measure. For example, suppose that three telecoms each purchased 20MHz of spectrum in region a, while two of the three also purchased 20MHz of spectrum in region b. In this case, the third firm may be viewed as less of a competitor by the other two. At the same time, from its perspective, the third firm may view these two firms as very strong competitors. Additionally, from the consumers point of view, it would appear that there are two firms really competing with each other (as opposed) to three since having access to spectrum in both region a and region b is important when deciding between phone and data packages.

For each package we calculate the weighted (over all other bidders) total overlap as

$$comp_j^N = \sum_{q \neq j} w_q C_{jq}. \tag{11}$$

In principle, different bidders could be given different weights. For simplicity, our empirical work assumes that $w_q = 1$ for all q.

Therefore, the set of variables that we will use to form various specifications of the structural profit function are:

$$x_J = \{\{pop_j\}_{j=1}^J, \{MHz_j\}_{j=1}^J, geo, geo_J^M, spec, comp_J^H, comp_J^N\}.$$

5 Results

We estimate both nonsmooth and smooth versions of the matching estimator. For the nonsmooth estimator we follow Bajari and Fox (2009) and use subsampling methods to obtain standard errors for our parameter estimates. For the smoothed maximum score matching estimator we deploy the Gaussian distribution function with bandwidth constructed following Horowitz (1992) as described earlier, which allows us to use bootstrapping methods to obtain the confidence intervals.

Before we present our empirical results, Table 2 provides a summary of bidder and winning package characteristics. Two bidders (13 and 15) won licenses covering Canada, though as the column geo^M indicates, bidder 13 won more total spectrum than bidder 15 — indeed, bidder 13 won 20MHz of spectrum nationwide, while bidder 15 won a combination of 10MHz and 20MHz licenses that covered the nation, reducing the extent of spectrum-weighted geographic complementarities. Also notice that the bidders captured a high percentage of geographic complementarities. Since some licenses went unsold, the maximum that geo^M (resp. geo) could take is 100.25 (resp. 7.006). As can be seen from the table, the actual allocation led to spectrum-weighted geographic complementarities of 94.37 (94.1% of the maximum possible), while unweighted geographic complementarities totaled 5.628 (80.3% of the maximum possible).

Bidders 13 and 15 represent two of the three incumbents, while the other incumbent is given

Table 2: SUMMARY OF BIDDER AND WINNING PACKAGE CHARACTERISTICS

Bidder	Status	$elig \sum MHz{\cdot}pop$	geo	geo^M	$spec$	$comp^H$	$comp^N$
1	Entrant	0.910	0.300	6.156	135.851	0.045	3.568
2	Entrant	0.163	0.033	1.168	45.652	0.004	2.000
3	Entrant	0.367	0.587	5.870	58.904	0.112	5.133
4	Entrant	1.485	0.301	10.642	431.820	0.060	2.434
5	Incumbent	0.937	0.866	12.783	214.707	0.153	4.290
6	Entrant	0.002	0.012	0.122	3.477	0.007	5.000
7	Entrant	0.063	0.096	2.596	101.467	0.021	2.858
8	Entrant	0.000	0.000	0.000	0.357	0.001	6.000
9	Entrant	0.267	0.438	4.828	67.106	0.080	5.693
10	Entrant	0.994	0.737	10.705	182.220	0.121	4.462
11	Entrant	0.037	0.228	2.278	22.951	0.037	5.762
12	Entrant	0.000	0.000	0.000	0.443	0.001	6.000
13	Incumbent	3.800	1.000	20.000	400.000	0.159	3.631
14	Entrant	0.013	0.028	1.113	52.026	0.003	1.500
15	Incumbent	1.413	1.000	16.114	283.410	0.171	3.984
TOTAL	—	10.450	5.628	94.375	2000.39	0.974	62.32

by bidder 5. In terms of capturing spectrum-weighted geographic complementarities, the three incumbents were the most successful. At the same time, there were at least four entrants who captured fairly large geographic footprints. Table 2 also highlights some apparent differences in entrant-bidder strategies. For example, comparing bidders 3 and 4, it appears that the former attempted to win less spectrum but spread over a larger geographic footprint than the latter. One result of this is that bidder 3 faces a more competitive landscape than does bidder 4.

5.1 Results Based on Maximum Score Estimation

Table 3 reports estimation results for a number of different specifications. Panel (a) restricts attention to unweighted geographic complementarities (*i.e.,* (5)). We report the estimate for the model with only unweighted geographic complementarities for comparison purposes only. We believe that this model is misspecified since it does not control for the amount of spectrum won by the bidders, either directly via *spec* or indirectly via our competition measures. Panel (b) provides results based on our spectrum-weighted measure of geographic complementarities (*i.e.,* (6)). In brackets, below each estimate, we report the 95% confidence interval, based on the subsampling method outlined above. For now, we focus only on the sign and significance of the estimated coefficients. Given the differences in the underlying auction and the support of our variables, a direct comparison of the coefficients with Bajari and Fox (2009) (or even across specifications) is not possible. Later, when we conduct a counterfactual analysis, a comparison with their results is possible.

Table 3: ESTIMATION RESULTS FOR THE MAXIMUM SCORE ESTIMATOR

(a) UNWEIGHTED GEOGRAPHIC COMPLEMENTARITIES

	(1a)	(2a)	(3a)	(4a)	(5a)	(6a)
$e \sum MHz \cdot pop$	1	1	1	1	1	1
geo	85.10 $[11.1, 749.2]$	5088.6 $[750.1, 8340.9]$	41.11 $[20.5, 248.9]$	62.24 $[42.57, 237.1]$	14.67 $[9.7, 33.0]$	82.02 $[3.26, 407.5]$
$spec$		6.93 $[0.719, 7.50]$		0.016 $[0.003, 0.053]$		0.07 $[-0.390, 0.141]$
$comp^H$			-180.7 $[-1003.7, -111.3]$	-237.5 $[-757.9, -161.3]$		
$comp^N$					-1.29 $[-2.9, -0.43]$	-0.90 $[-38.12, 0.106]$
Fit	0.8838	0.9308	0.9521	0.9575	0.9322	0.9311

(b) SPECTRUM-WEIGHTED GEOGRAPHIC COMPLEMENTARITIES

	(1b)	(2b)	(3b)	(4b)	(5b)	(6b)
$e \sum MHz \cdot pop$	1	1	1	1	1	1
geo^M	2.47 $[1.4, 5.5]$	55.92 $[7.08, 960.2]$	25.48 $[1.3, 62.5]$	16.69 $[5.72, 61.17]$	13.05 $[2.7, 19.4]$	33.26 $[2.19, 219.4]$
$spec$		0.19 $[0.016, 0.737]$		0.028 $[0.005, 0.084]$		0.09 $[-0.079, 0.215]$
$comp^H$			-55.30 $[-105.4, 1.9]$	-104.5 $[-257.1, -33.40]$		
$comp^N$					-0.50 $[-1.1, 1.1]$	-0.59 $[-25.59, 1.48]$
Fit	0.9449	0.9464	0.9454	0.9558	0.9480	0.9482

95% confidence intervals in brackets below each estimated coefficient.

The first thing to notice is that, both measures of geographic complementarities are always significantly positive, though the estimates themselves vary substantially depending on the model under consideration. The fact that the coefficients on both *geo* and geo^M vary across specifications is likely due to the different scales of the other variables.

Look next at the estimation results in which we include either of our two measures of competition. In all cases, the estimated coefficient is negative, as we would expect. Our Herfindahl-based measure of competition is significant in 3 of the 4 specification in which it enters, while our measure of competition based on niche overlap is only significant in specification 5(a). Comparing the two measures of competition, we see that the Herfindahl-based measure generally fits the data better than does the niche overlap measure of competition.

One of our questions above concerned the way in which the amount of spectrum won by a bidder enters into the structural profit function. In particular, does the amount of spectrum won enter interact with geographic complementarities, does it have a separate effect, or possibly both? To gain some insight, consider specifications (1a), (1b), (2a) and (2b). Recall that specification (1a), which replicates Bajari and Fox (2009), does not control for the amount of spectrum won either directly or indirectly. Given that the fit is so much lower here than in any other case, it is clear that (1a) lacks explanatory power. If we add the amount of spectrum directly by including *spec*, as in (2a), the fit improves by about 5 percentage points, and spec is found to be significant. On the other hand, if we control for the amount of spectrum indirectly via spectrum-weighted geographic complementarities, as in (2a), then geo^M is significant and the fit improves by 6.1 percentage points. Thus, the amount of spectrum won is important. Moreover, as (2b) suggests, it appears that the amount of spectrum won has a direct effect via *spec*, as well as an indirect effect via geo^M. In both cases, the effect is positive, meaning that profits increase the more spectrum a bidder has. Although the fit in (2b) does not increase by much, relative to (1b), it is still the best-fitting model of the four considered. Moreover, the variable *spec* remains significant at the 5% level even after controlling for spectrum-weighted complementarities.

Overall, these results affirm that the amount of spectrum a bidder wins has a strong affect on its profits. However, just by looking at the coefficients, it is difficult to get a sense of how big of an impact each of these variables has. For example, if a bidder wins more spectrum in a given region then, geo^M and *spec* will increase, while both $comp^H$ and $comp^N$ will decrease, leading, in all cases, to an increase in profits. However, we would like to understand the relative contribution of each variable. In our counterfactual analysis, below, we will attempt to get at precisely this.

5.2 Results Based on Smoothed Maximum Score Estimation

The results for our smoothed maximum score estimations are presented in Table 4. We highlight the similarities and differences between these results and the maximum score estimates presented above. As can be seen, in all cases the estimated coefficients have the same sign as in our maximum score estimates. Moreover, in all cases both *geo* and geo^M remain significantly different from zero.

Table 4: ESTIMATION RESULTS FOR THE SMOOTHED MAXIMUM SCORE ESTIMATOR

(a) UNWEIGHTED GEOGRAPHIC COMPLEMENTARITIES

	(1a)	(2a)	(3a)	(4a)	(5a)	(6a)
$e\sum MHz\cdot pop$	1	1	1	1	1	1
geo	90.86 $[87.3, 99.0]$	1.38E+05 $[52722, 1.108e8]$	15.26 $[11.4, 1303]$	544 $[332, 37233]$	11.12 $[1.57, 7.81]$	777.9 $[329, 3076]$
spec		183.7 $[66.2, 1.51e5]$		0.126 $[0.08, 8.27]$		0.936 $[0.32, 3.63]$
$comp^H$			-66.82 $[-6289, -50.4]$	-2090 $[-1.57e5, -1304]$		
$comp^N$					-0.94 $[-0.79, -0.14]$	-4.77 $[-25.0, -2.42]$
Fit	0.8864	0.9273	0.9481	0.9533	0.9275	0.9285

(b) SPECTRUM-WEIGHTED GEOGRAPHIC COMPLEMENTARITIES

	(1b)	(2b)	(3b)	(4b)	(5b)	(6b)
$e\sum MHz\cdot pop$	1	1	1	1	1	1
geo^M	10.32 $[0.97, 15.5]$	742 $[416, 1.38e7]$	16597 $[44.3, 477085]$	7880.7 $[942, 2.29e7]$	139.7 $[59.2, 165.7]$	248.2 $[188, 1299]$
spec		2.5044 $[1.41, 45851]$		14.76 $[1.90, 38637]$		0.706 $[0.52, 3.93]$
$comp^H$			-35212 $[-1045827, -59.3]$	-47754 $[-1.52e8, -6318]$		
$comp^N$					-6.05 $[-7.51, -3.27]$	-4.35 $[-23.3, 3.22]$
Fit	0.9410	0.9432	0.9420	0.9528	0.9443	0.9442

95% confidence intervals in brackets below each estimated coefficient.

There are two notable differences between our maximum score and our smoothed maximum score estimates. First, $comp^H$ becomes significant in specification 3(b), while $comp^N$ becomes significant in specifications 6(a) and 5(b), while *spec* also becomes significant in specification 6(a). Thus, the effect of competition would appear to be stronger, even after controlling for spectrum.

Second, the magnitude of the estimated coefficients is generally substantially greater in the smoothed maximum score case. Interesting, however, is the fact that, except for specification 6(a), the ratio of the coefficients are approximately the same. That is,

$$\frac{\beta^{MS}_{comp}}{\beta^{MS}_{geo}} \approx \frac{\beta^{SMS}_{comp}}{\beta^{SMS}_{geo}}$$

across all specifications, and similarly when comparing *geo* with *spec* as well as *comp* and *spec*. What this implies is that the relative importance of competition to complementarities is constant across the two estimation methods, while the relative importance of each variable with respect to initial eligibility is larger in the smoothed maximum score setting. The reason for this is simple. With the intercept fixed at 1 and for a given bandwidth, the estimated coefficients will change, thus, the relationship between each slope coefficient and the intercept will undoubtedly change (in this case increase). The fact that the relative change is identical across the two models though suggests that our remaining findings are robust across both estimation methods.

5.3 Counterfactual Analysis

In order to understand the efficiency properties, and in particular, the impact of the set-aside, of this auction three different counterfactuals. First, we consider by what percentage the structural profit function (summed over all winning bidders) would have changed if, given our estimated parameters, the bidder with the highest initial eligibility (*i.e.,* Bidder 13; a.k.a. Rogers) won all of the licenses. This allocation will be the one that maximizes $e \sum MHz \cdot pop$, geo^M and *spec*, while also leading to minimal competition.

Second, we consider by what percentage the structure profit function (summed over all winning bidders) would have changed if the three incumbents divided all of the spectrum roughly proportional to their initial eligibilities. This represents what we feel to be the most likely outcome in the absence of a set-aside. Finally, we consider by what percentage the structural profit function (summed over all winning bidders) would have changed if the eight largest bidders in terms of initial eligibility won each of the eight blocks nationwide.[23] This assignment can be argued to be efficient from the point of view of awarding the most spectrum nationwide to those firms with the highest eligibility. This will also lead to the maximal value of *geo*. Both $e \sum MHz \cdot pop$ and geo^M should increase, though they will not be maximal. Finally, the effect on *spec* and competition is, *a priori*, ambiguous.

Before doing this, however, we report in Table 5 the package characteristics of the three counterfactual scenarios. Panel (a) contains results for the first counterfactual scenario (*i.e,* a single winner), while panel (b) contains the results of the second counterfactual scenario (*i.e.,* the three incumbent bidders winning) and panel (c) contains the results of our third counterfactual scenario

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[23] Those bidders with the highest eligibility were allocated a 20MHz license, while those with less were allocated a 10 MHz license and, finally, the bidder with the 8th highest initial eligibility won the 5MHz I block. Licenses that went unsold in the actual auction were excluded from the counterfactual analysis.

(*i.e.,* the eight largest bidders winning). Finally, panel (d) summarizes the package characteristics based on the actual allocation in the auction. As can be seen, going from the actual allocation to a scenario in which the eight largest bidders won increases $e \sum MHz \cdot pop$ by approximately 23% and geo^M by a more modest 5.6%. Furthermore, while our Herfindahl-based measure of competition is little changed, our niche overlap-based measure of competition is reduced by nearly one-third. This is consistent with our measures of competition since this counterfactual analysis would suggest much less overlap in the counterfactual setting while concentration would stay roughly the same (since the MHz concentration is proportioned equally in each of the 59 regions). Next, comparing the three counterfactual scenarios, we see that there is very little difference in total spectrum-weighted geographic complementarities, a larger difference in $e \sum MHz \cdot pop$, while by far the largest effect is that both our competition measures are reduced to 0, meaning that a single winner actually faces no competition, while *spec* is an order of magnitude higher.

In Table 6 we report the results of our counterfactual study. We report results for both the maximum score and smoothed maximum score and for each of our 12 different specifications. To ease the comparison, for each specification we normalize the estimated profits (at the actual auction allocation) to 100; this is the second column. The third column reports profits, relative to the actual auction, if a single winner won all licenses, while the fourth column reports profits, relative to the actual auction, if the three incumbents won all licenses, and the fifth column reports profits, relative to the actual auction, if the eight largest bidders each won an entire spectrum block. The sixth column shows the largest percentage increase in profits. The final column shows the contribution made by each variable to the maximal possible efficiency gain.

Before getting into the specifics, we first note that, with one exception, the size of the efficiency gains are very similar between the maximum score and smoothed maximum score estimators. This is despite the vastly different coefficient estimates. Note also that the contribution of $e \sum MHz \cdot pop$ is lower in our smoothed maximum score counterfactuals. This is consistent with our earlier intuition that the relative importance of competition to complementarities is constant across the two estimation methods, while the relative importance of each variable with respect to initial eligibility is larger in the smoothed maximum score setting.

When judging whether the auction was successful, Table 6 suggests that the answer depends crucially on whether one believes that geographic complementarities are best captured by *geo* or by geo^M. Leaving aside specification 5(a), which would seem to be clearly implausible, the efficiency gain using unweighted geographic complementarities ranges from 45.6 to 91.1%, while when using spectrum-weighted complementarities, the range is from 9.0 to 32.9%. The former case leads to results which are roughly similar to Bajari and Fox (2009), and would indicate an inefficient auction.

As we have said above, we feel that there is a compelling case to be made that spectrum-weighted geographic complementarities is the appropriate measure. For example, look back at Table 2 and consider bidders 13 and 15. While both won licenses covering the entire country (hence, $geo = 1$

Table 5: PACKAGE CHARACTERISTICS UNDER COUNTERFACTUAL SCENARIOS[†]

(a) SINGLE BIDDER WINS

Bidder	Status	$elig \sum MHz \cdot pop$	geo	geo^M	$spec$	$comp^H$	$comp^N$
13	Incumbent	19.05	1	100.25	10053	0	0

(b) BIG 3 WIN

Bidder	Status	$elig \sum MHz \cdot pop$	geo	geo^M	$spec$	$comp^H$	$comp^N$
5	Incumbent	2.12	1	30	900.0	0.250	2.000
10	Incumbent	2.70	1	30.75	948.8	0.245	1.967
13	Incumbent	7.50	1	39.50	1564.7	0.184	1.532
TOTAL	—	12.32	3	100.25	3413.5	0.679	5.499

(c) ASSORTATIVE MATCHING

Bidder	Status	$elig \sum MHz \cdot pop$	geo	geo^M	$spec$	$comp^H$	$comp^N$
3	Entrant	0.0467	0.085	0.424	3.75	0.023	6.664
5	Entrant	0.6716	0.922	9.216	94.95	0.141	6.052
15	Incumbent	0.877	1	10	100	0.149	6.025
10	Incumbent	0.8921	1	10	100	0.149	6.025
4	Entrant	1.1693	1	10	100	0.149	6.025
2	Entrant	2.4923	1	20	400	0.119	4.012
1	Entrant	2.9007	1	20	400	0.119	4.012
13	Incumbent	3.800	1	20	400	0.119	4.012
TOTAL	—	12.85	7.006	99.64	1598.70	0.970	42.83

(d) OVERALL ACTUAL PACKAGE CHARACTERISTICS (FROM TABLE 2)

Bidder	Status	$elig \sum MHz \cdot pop$	geo	geo^M	$spec$	$comp^H$	$comp^N$
TOTAL		10.450	5.628	94.375	2000.39	0.974	62.32

[†] Note that in the auction only 282 of 292 licenses were sold, with the G and I blocks having unsold licenses. Our analysis here assumes that this carries through under our various counterfactual scenarios. This is why geo^M, for example, does not sum to 105 in panels (a) and (b) of the table.

Table 6: Counterfactual Analysis

(a) Based on Table 3 (maximum score)

Specification	Est. Profit (Normalized)	Single Winner	Big 3	Assortative Matching	Largest % gain	Contribution of $(elig, geo, spec, comp)$[†]
1(a)	100	36.60	60.70	124.31	24.3%	$(.11, .89, -, -)$
2(a)	100	175.83	91.57	109.96	75.8%	$(0, -.73, 1.73, -)$
3(a)	100	91.53	19.63	191.05	91.1%	$(.04, .95, -, .01)$
4(a)	100	148.87	56.87	151.54	51.5%	$(.03, 1.03, -.08, .01)$
5(a)	100	271.65	396.50	485.26	385.3%	$(.05, .42, -, .53)$
6(a)	100	145.64	88.77	118.77	45.6%	$(.03, -1.49, 2.24, .22)$
1(b)	100	109.49	106.72	106.32	9.5%	$(.37, .63, -, -)$
2(b)	100	132.92	110.56	103.89	32.9%	$(0, .18, .82, -)$
3(b)	100	108.98	107.11	105.79	9.0%	$(.04, .71, -, .25)$
4(b)	100	128.18	111.06	105.16	28.2%	$(.02, .23, .52, .23)$
5(b)	100	109.59	108.82	106.67	9.6%	$(.07, .66, -, .27)$
6(b)	100	130.16	111.00	104.58	30.2%	$(.01, .2, .76, .04)$

(b) Based on Table 4 (smoothed maximum score)

Specification	Est. Profit (Normalized)	Single Winner	Big 3	Assortative Matching	Largest % gain	Contribution of $(elig, geo, spec, comp)$[†]
1(a)	100	33.89	59.64	124.34	24.3%	$(.09, .91, -, -)$
2(a)	100	173.46	90.98	110.18	73.5%	$(0, 1.63, -.63, -)$
3(a)	100	76.72	5.46	200.17	100.2%	$(.01, .98, -, .01)$
4(a)	100	141.77	50.77	155.22	55.2%	$(0, 1.05, -.07, .01)$
5(a)	100	261.78	611.28	811.45	711.4%	$(0, .45, -, .54)$
6(a)	100	171.17	92.48	113.27	71.2%	$(0, 1.35, -.48, .12)$
1(b)	100	106.27	106.23	105.59	6.3%	$(.01, .99, -, -)$
2(b)	100	132.69	110.52	103.87	32.7%	$(0, .18, .82, -)$
3(b)	100	108.60	107.04	105.71	8.6%	$(0, .74, -, .26)$
4(b)	100	129.12	111.17	104.92	29.1%	$(0, .22, .56, .22)$
5(b)	100	109.37	109.09	106.67	9.4%	$(0, .68, -, .31)$
6(b)	100	130.20	111.00	104.52	30.2%	$(0, .20, .77, .04)$

[†] Numbers should sum to 1, modulo rounding.

Highlighted cells represent the counterfactual scenario which leads to the greatest increase in profits for the specification.

for both), the unweighted measure fails to capture the fact that bidder 13 won 20MHz nationwide, while bidder 15 won a combination of 10MHz and 20MHz licenses. Therefore, while bidder 13 can provide a consistent level of service across the entire country, bidder 15's service offerings may be limited in those regions where it only won 10MHz of spectrum, which should lead to lower complementarities. The spectrum-weighted measure, geo^M, captures precisely this intuition.

Therefore, if we focus on only those specifications which use spectrum-weighted geographic complementarities, 1(b)–6(b), then it would appear that the auction was not wildly inefficient. That being said, there is still a large difference depending on whether or not one separately controls for the amount of spectrum won via *spec*. If we exclude *spec* from our counterfactual, then the efficiency gain is approximately 9.5% (or approximately \$400 million on a \$4.25 billion auction). On the other hand, if we include *spec*, then the efficiency gain is approximately 30% (or \$1.28 billion).[24] Of this, notice that between 52 and 82% of the gain is due to the dramatic increase in the value of *spec* going from the actual allocation to the counterfactual allocation of a single winner. The increase in geo^M is responsible for another 18 to 23% of the efficiency gain. The remaining portion is picked up by the increase in $e \sum MHz \cdot pop$ and the reduction in competition.

While a single bidder winning all of the spectrum may represent the upper bound on auction efficiency, it is also a very unlikely outcome even in the absence of the auction set-aside. As we said above, we believe that the most plausible outcome in the absence of the auction set-aside (or some other mechanism, such as a spectrum cap, to limit the ability of incumbents to hoard spectrum) is that the "Big 3" incumbents would divide the spectrum amongst themselves. This counterfactual scenario would lead to an efficiency gain of between 6.7 and 11.1%, or approximately \$471 million at the upper bound of this range. While by no means a small number, it is easy to think of scenarios in which the effect on prices or services from new entry could offset this cost in relatively short order. For example, in 2006, it was estimated that there were approximately 18.6 million cell phones in use in Canada. If prices drop by an average of \$1 per cell phone per month, the effect of the loss in auction efficiency will be offset in only two years.

6 Conclusions

Prior to Industry Canada's AWS auction, a stated goal was to increase competition in the wireless phone industry. To achieve this goal Industry Canada employed a set-aside policy to prevent incumbents from bidding in all 8 blocks of available spectrum. Set-aside auctions are one of a cluster of regulatory policies available to auctioneers who may wish to achieve certain "desirable" outcomes. Unfortunately, even with this appeal, very little empirical work exists exploring just how beneficial these policies are, and even less structural work exists in this arena. Our study is one of

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[24] Even this amount is likely over-stated because it fails to take into account that the other two incumbents already have spectrum in other ranges. Therefore, while competition would be reduced, it would not disappear entirely, as this counterfactual suggests.

the first to structurally investigate the ramifications of such a policy for auctioning spectrum. We use recently developed pairwise matching estimators and the notion of pairwise stability to estimate the parameters of the telecommunications firms who bid in the spectrum auctions profit function. These estimates were then used to consider alternative spectrum allocation schemes to determine the efficacy of the set-aside for Industry Canada's stated goals of increasing competition.

Our results suggest that auction revenue may have increased by as much as \$1.28 billion under alternative scenarios. While this is an upper bound, it was obtained under the assumption that a single bidder won all of the available spectrum. More realistically, in the absence of the set-aside, a likely outcome is that the three incumbents would win all of the spectrum. In this case, our results suggest an efficiency loss on the order of \$400 − 471 million. As we have argued, while this is a non-negligible amount of money, it is certainly plausible that enhanced competition could lead to consumers benefiting by more than this amount. Indeed, as of the end of 2010, there are 4 entrants providing wireless services in throughout Canada using their own networks, with more entry expected in 2011 and 2012.

Certainly many Canadians now have, or will have, more choices available to them when deciding amongst phone and data plans. It remains to be seen whether competition takes off and continues in a lasting manner unlike Canada's previous attempt to promote competition. In 1995, two new entrants were awarded 30MHz of PCS spectrum; however, competition was short-lived as one of the new entrants was purchased by Telus in 2000, and the other was purchased by Rogers in 2004. At the end of 2010, three new entrants had collectively captured over 340,000 subscribers, while Québecor began providing services with its own network, rather than as an mobile virtual network operator (MVNO) with Rogers.[25]

From an econometric standpoint this paper deployed both the maximum score and the smoothed maximum score estimators in concert. We are unaware of empirical studies that have used both estimation routines in unison. Our results suggest that qualitatively both estimators provide nearly identical insights regarding our counterfactual analyses, but given the lack of a formal bandwidth selection mechanism, the differences in our confidence intervals across estimators merits room for further investigation. Additionally, given that the smooth maximum score estimator does not have an advantage over the maximum score estimator unless sufficiently many derivatives of the underlying primitives exist, it is important to compare results in any empirical analysis deploying these estimators.

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[25] See Footnote 5, as well as the article, "Wireless upstarts score on new subscribers," *The Globe and Mail*, 18 December 2010.

References

Ayres, I., and P. Cramton (1996): "Deficit reduction through diversity: How Affirmative Action at the FCC increased auction competition," *Stanford Law Review*, 48, 761–815.

Bajari, P., and J. Fox (2009): "Measuring the Efficiency in an FCC Spectrum Auction," Working Paper.

Cramton, P., A. T. Ingraham, and H. J. Singer (2008): "The Effect of Incumbent Bidding in Set-Aside Auctions: An Analysis of Prices in the Closed and Open Segments of FCC Auction 35," *Telecommunications Policy*, 32, 273–290.

Dubois, P., M. Ivaldi, and T. Magnac (2008): "Introduction to the special issue on the Econometrics of Auctions," *Journal of Applied Econometrics*, 23, 867–869.

Fox, J. (2009): "Estimating Matching Games With Transfers," Working Paper.

Hall, P. (1992): *The Bootstrap and Edgeworth Expansion.* New York: Springer.

Han, A. K. (1987): "Nonparametric analysis of a generalized regression model: The maximum rank correlation estimator," *Journal of Econometrics*, 35, 303-316.

Hazlett, T. W., and R. E. Muñoz (2009): "Spectrum Allocation in Latin America: An Economic Analysis," *Information Economics and Policy*, 21, 261–278.

Horowitz, J. (1992): "A Smoothed Maximum Score Estimator for the Binary Response Model," *Econometrica*, 60, 505–531.

——— (2002): "Bootstrap Critical Values for Tests Based on the Smoothed Maximum Score Estimator," *Journal of Econometrics*, 111, 141–167.

Industry Canada (2007): "Policy Framework for the Auction for Spectrum Licenses for Advanced Wireless Services and other Spectrum in the 2GHz Range," Discussion paper, Industry Canada.

Jack, Q., and T. Suri (2011): "Mobile Money: The Economics of M-PESA," NBER Working Paper 16721.

Manski, C. F. (1975): "Maximum score estimation of the stochastic utility model of choices," *Journal of Econometrics*, 3, 208–228.

Politis, D. N., and J. P. Romano (1994): "Large Sample Confidence Region Based on Subsamples Under Minimal Assumptions," *The Annals of Statistics*, 22(4), 2031–2050.

Sohn, M.-W. (2002): "A Relational Approach to Measuring Competition Among Hospitals," *Health Services Research*, 37(2), 457–482.

Waverman, L., M. Meschi, and M. Fuss (2005): "The impact of telecoms on Economic growth in developing countries," *Vodafone Policy Paper Series.*